

# Schwarzschild's Method for Constructing Galaxy Models



# Dynamical Models

- Aim: find phase-space distribution function  $f$ 
  - Provides orbital structure
  - Mass-density distribution  $\rho = \iiint f d^3v$
  - Velocities  $v$  derive from gravitational potential  $V$
  - *Self-consistent* model:  $4\pi G\rho = \nabla^2 V$
- Approaches
  - Assume  $f$  find  $\rho$  (but what to assume for  $f$ ?)
  - Assume  $\rho$  find  $f$  (solve integral equation)
- Use Jeans theorem  $f = f(I)$  to make progress
  - Provides  $f(E, L)$  for spheres,  $f(E, L_z)$  for axisymmetry
  - $f(E, I_2, I_3)$  for *separable* axisymmetric & triaxial models

# Spheres

- Mass model
  - Defined by density profile  $\rho(r)$
- Gravitational potential  $V(r)$  by two single integrations
- Hamilton-Jacobi equation separates in  $(r, \theta, \varphi)$ 
  - Four integrals of motion:  $E, L_x, L_y, L_z$
  - All orbits regular: planar rosette's
- Self-consistent models
  - Isotropic models  $f=f(E)$  via Abel inversion (Eddington 1916)
  - Circular orbit model: only orbits with zero radial action
  - Many distribution functions:  $f=f(E), f=f(E+aL), f(E, L)$ , corresponding to different velocity anisotropies
  - Constrain  $f$  further by measuring kinematics

# Triaxial Galaxies

- Observational evidence for triaxial systems
  - Bulge of M31 triaxial
    - Contopoulos 1956, Stark 1977
  - Surface photometry of ellipticals
    - PA of major axis of isophotes varied with radius: triaxial shape
    - Martin carried out his own observations with Ted Williams in 1976
  - Slow rotation of giant ellipticals caused by anisotropic velocity dispersions and hence natural shape is triaxial
    - Binney 1976
- These systems seemed to be in steady state
  - There was very little understanding of the dynamics
  - This clearly motivated Martin's interest

# Letter to Maria Teresa Ruiz 23 Aug 1978

Now an abbreviated report from here; the Copernicus satellite continues to operate fantastically well in spite of being now six years in orbit, which keeps Jack Rogerson, Ed Jenkins and Don York very busy through their attempts together with our television engineers to get new NASA contracts for new grants for new space experiments ~~and~~ thus far not had concrete results. One year from now Lyman Spitzer will resign from the Chairmanship of our department and Jerry Ostriker will take it over. You probably know that Richard Harm retired one year ago and that at that time according to previous plans I have stopped working in the stellar interior and have since completely concentrated on a dynamics problem for tri-axial elliptical galaxies which I much would enjoy discussing with you. Ted Williams, with a bit of participation from me, has now reduced our Cerro Tololo observations on elliptical galaxies, as well as some additional ones we got last year on Mt. Lemmon with the result that twists in the inner portions of elliptical galaxies seem not at all so rare -as Ivan King's data had already indicated. This point is of particular interest for my theoretical work which really got started through your thesis investigation here.

For today then my very best wishes,

Sincerely,

*Martin Schwarzschild.*

Martin Schwarzschild

# Non-Spherical Models - Seventies

- Oblate and prolate axisymmetric models
  - Construction of  $f(E, L_z)$  for few special  $(\rho, V)$  pairs
    - General method to compute  $f(E, L_z)$ : Hunter & Qian (1993)
    - Model with Ruiz for M31 bulge was approximately  $f(E, L_z)$
  - It was known from numerical integrations that most orbits have a third 'effective' integral of motion  $I_3$ 
    - Ollongren (1962) showed that numerically computed orbits do not fill the space allowed by conservation of only  $E$  and  $L_z$
    - Contopoulos (1960) had derived series expansions for  $I_3$
    - Little attention paid to Kuzmin's (1956) work on separable models
- Triaxial shapes
  - Very little work:  $E$  only known integral
  - Kuzmin's ultra-short conference paper of 1973 on separable triaxial models remained unknown in West

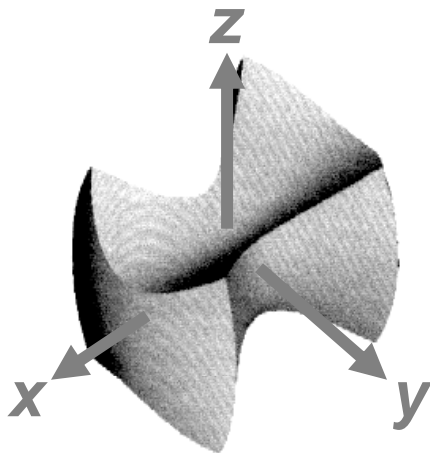
# Martin's Approach (1)

- Martin picked a simple triaxial potential-density pair
  - With a core: central cusps still to be discovered by HST
- Numerically calculated orbits
  - Most do not fill volume allowed by conservation of energy: two extra integrals  $I_2$  and  $I_3$  which define orbital shape
  - Three main families or orbits (shapes)
    - Box orbits, short-axis tubes and long-axis tubes
    - No intermediate-axis tubes
    - Tube orbits carry net angular momentum
  - Related to structure of main stable periodic orbits
  - Very modest fraction of stochastic orbits

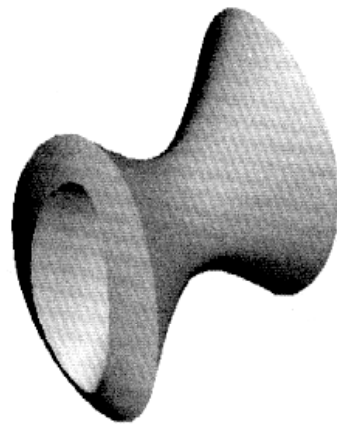
# Separable Triaxial Models

- Four orbit families

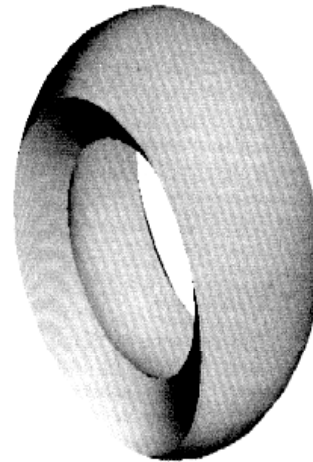
- Integrals  $I_2$  and  $I_3$  exact; no stochastic orbits



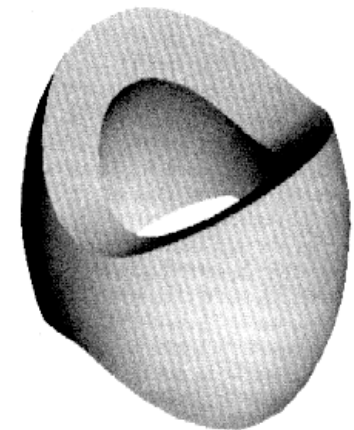
*Box*



*Inner long-axis tube*



*Outer long-axis tube*



*Short-axis tube*

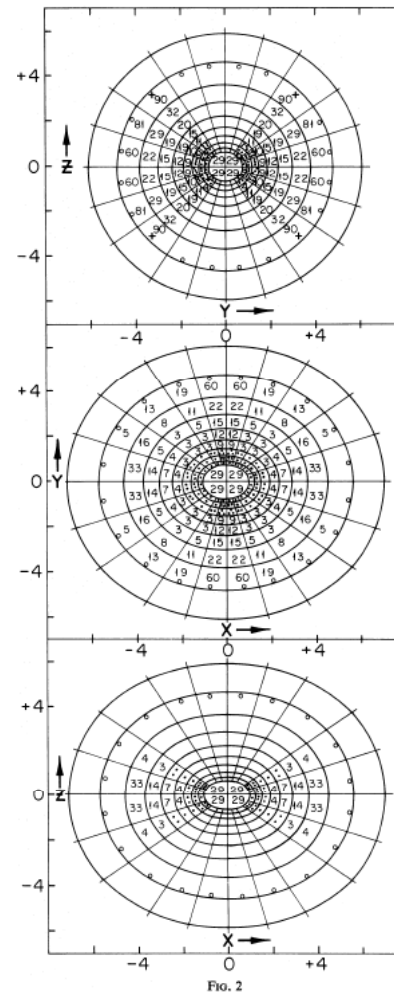
- Same *four* orbit families found in Martin's model

- Long-axis tubes come in two distinct shapes
- Confirmed in spring 1983

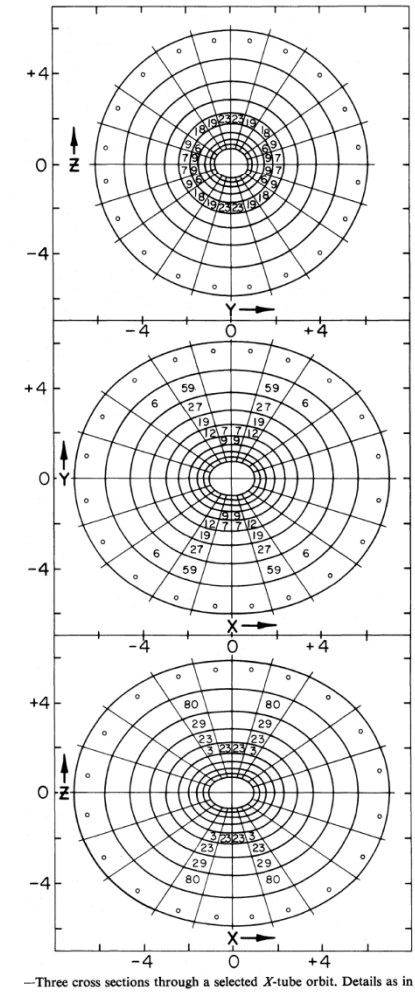


# Martin's Approach (2)

- Compute time-averaged orbital densities on grid
- Add orbital densities with non-negative weights so as to reproduce model density on grid
- This worked and is equivalent to 'finding f'
  - Distribution of velocities known at each point
  - Without knowing  $I_2$  and  $I_3$



Box

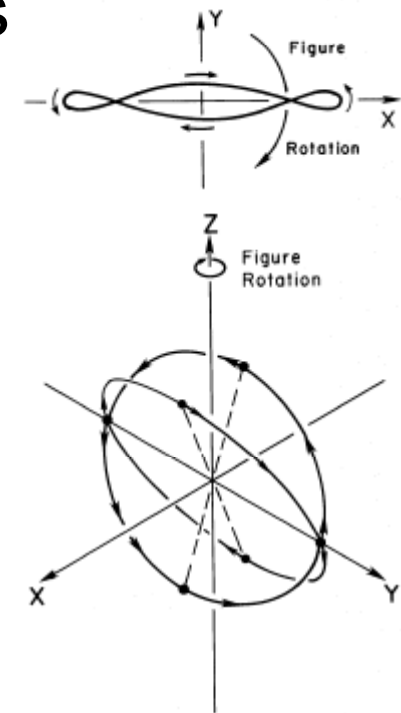


—Three cross sections through a selected X-tube orbit. Details as in Fig. 2.

Long-axis tube

# Tumbling Shapes

- Martin also applied his approach to a model with figure rotation about short axis
  - Slowly rotating giant elliptical
- Orbit structure rich: seven main families
  - Boxes acquire net rotation
  - Direct/retrograde tubes distinct
  - Direct/retrograde long-axis tubes tilt out of (Y, Z) plane differently
    - This means that in a triaxial galaxy they must be populated equally
    - No net streaming about long axis
- Little follow-up of this work
  - Vietri 1986 and Zhao 1996 for Galactic Bulge



# Cusped Models

- Observational evidence accumulated that ellipticals have central density cusps rather than cores
- Implications for orbital structure
  - Three tube orbit families remain
  - Box orbits replaced by mix of stochastic orbits and minor families (boxlets) related to higher-order resonances
    - Bananas, fish, anti-fish, pretzels, anti-pretzel,...
  - No *global* integrals  $I_2$  and  $I_3$
- Martin attempted to construct cusped model (1993)
  - Concluded that it would evolve slowly
  - Further work by Kuijken, Merritt, Valluri and others
- Supermassive central black hole has similar effect

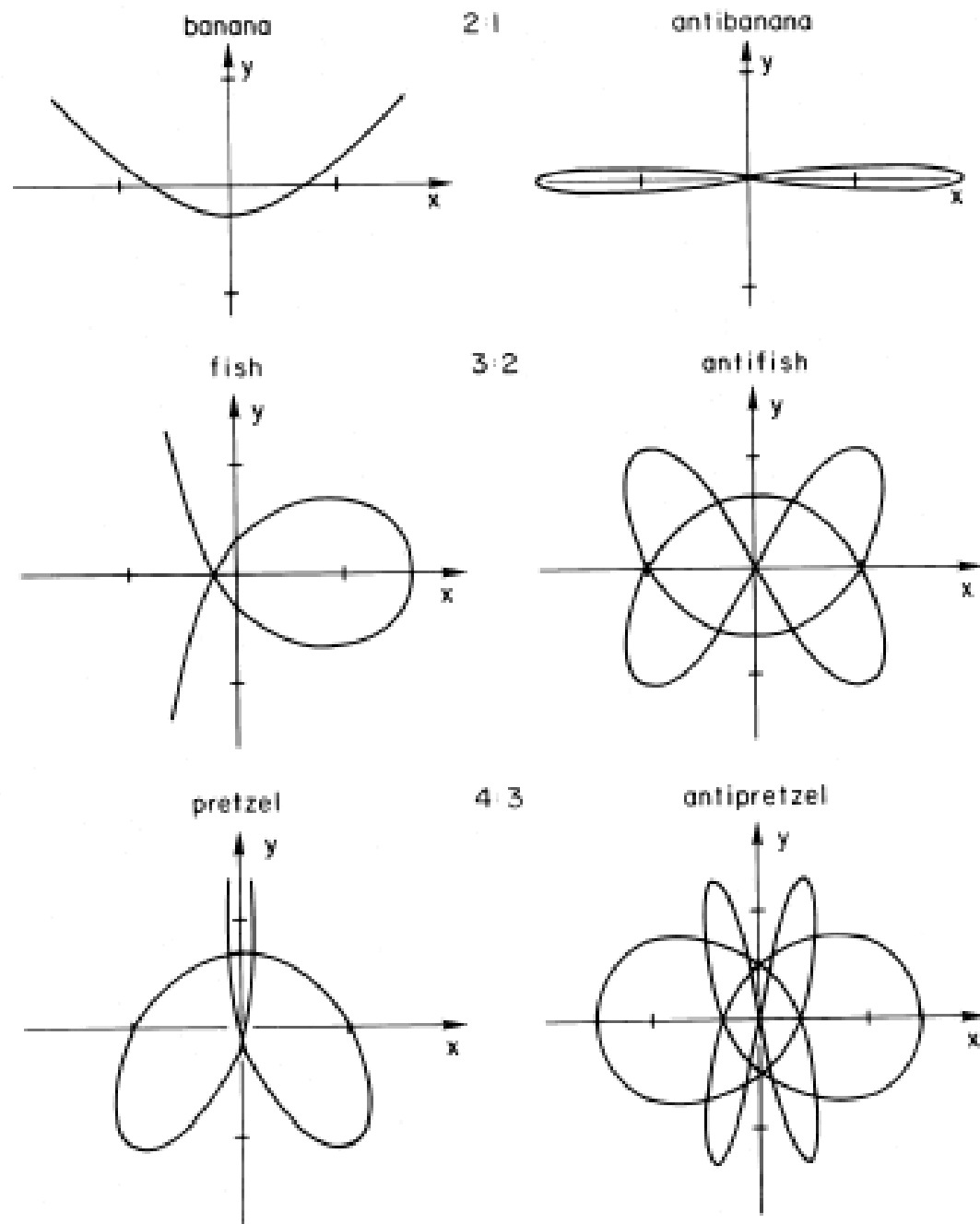


FIG. 4.—Closed boxlets in the singular logarithmic potential with axis ratio  $b = 0.7$ . *Left*: centrophobic (stable). *Right*: centrophilic (unstable).

# Power of the Method

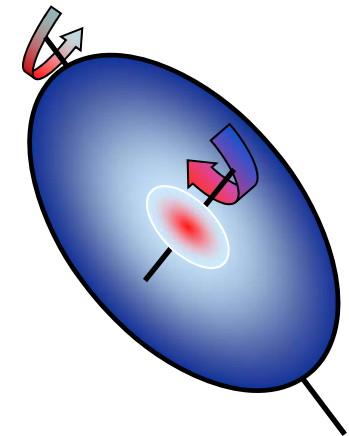
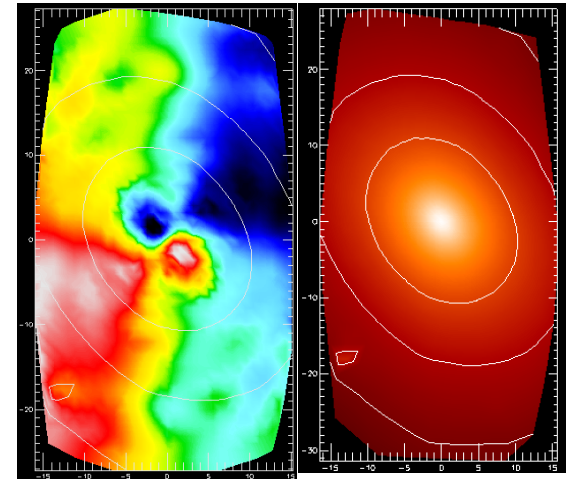
- No restriction on form of potential
  - Arbitrary geometry
  - Multiple components (BH, stars, dark halo)
- No restriction on distribution function
  - No need to know analytic integrals of motion
  - Full range of velocity anisotropy
- Possible to include all kinematic observables
  - Fit model directly to data on sky plane
  - Today multiple codes exist to do this for spherical, axisymmetric and non-tumbling triaxial geometry
  - Jalali & Tremaine (2011) show how to obtain smooth rather than discrete distribution functions with this method

# Applications

- Existence of equilibrium models
  - Depending on shape and strength of central cusp
- Measurement of central black hole masses
  - Fit photometry and absorption-line kinematics
  - Parameters: shape, stellar M/L and  $M_{\text{BH}}$
- Internal dynamical structure of giant ellipticals
  - Anisotropy and multiple components
  - Can be linked to properties of stellar populations
- Analysis of tumbling triaxial systems possible
  - Bars tumble and are 'ferociously triaxial'
  - Large number of parameters and rich orbital structure

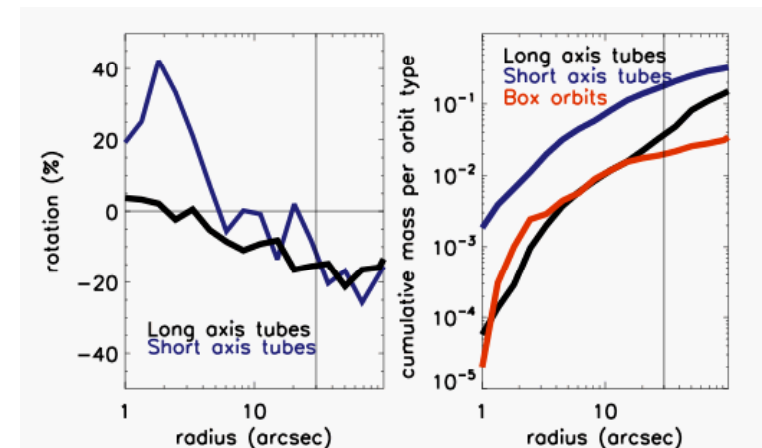
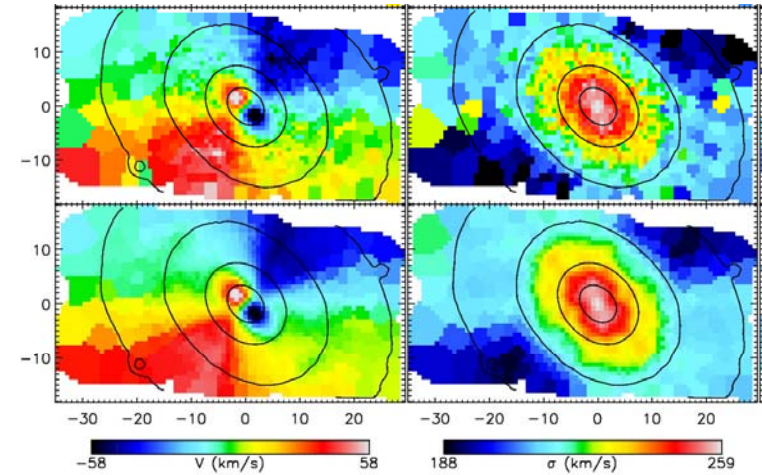
# The E3 Galaxy 4365

- Kinematically decoupled core
  - Long-axis rotator, core rotates around short axis
- SAURON kinematics:
  - Rotation axes of main body and core misaligned by  $82^\circ$
  - Consistent with triaxial shape, both long-axis & short-axis tubes occupied
- Traditional interpretation
  - Core is distinct, and remnant of last major accretion  $\sim 12$  Gyr ago



# Triaxial Dynamical Model

- Parameters
  - Two axis ratios, two viewing angles,  $M/L$ ,  $M_{\text{BH}}$
- Best-fit model
  - Fairly oblate (0.7:0.95:1)
  - Short axis tubes dominate, but  $\sim 50\%$  counter rotate, except in core; cf NGC4550
  - Net rotation caused by long-axis tubes, except in core
  - KDC *not* a physical subunit, but appears so because of embedded counter-rotating structure



*R. van den Bosch et al. 2008*



# Application to M31?

- Schwarzschild's method in principle able to construct dynamical model for M31
  - But many parameters to be chosen/explored
- Made-to-measure particle models may fare better
  - Proposed by Syer & Tremaine 1996
  - Adapt particle weights in an N-body system until observables are well-matched
  - Allowing easier analysis of stability and evolution
  - Recent implementation: NMAGIC by Gerhard & Morganti
  - Should allow detailed modeling of M31, much like earlier MW model by Gerhard & Bissantz 2004
- Interaction history limits value of equilibrium model

# Conclusions

- Martin's 'quiet retirement' in fact transformed the field by opening a practical way to model galaxies
  - Triggered by new observations, but made possible by clear thinking leading to a non-traditional approach
  - Another example of the process described in his preface to 'Structure and Evolution of the Stars'
- He involved a succession of Princeton graduate students in different aspects of this research
  - Heiligman, Goodman, Heisler, Merritt, Vietri, Ratcliff, Chang, Statler, Park, Miralda-Escudé and Lees
- And influenced many others to this day