1. The black hole at the centre of M31

(just P3)
Distinct component in UV

Scale: P1-P2 separated by $\sim 0.5'' \sim 2$ pc.
Photometry of P3
(Bender et al 2005)

Distinct component in UV

Scale: P1-P2 separated by $\sim 0.5'' \sim 2$ pc.
Distinct from surrounding P1–P2 eccentric disc:

- 100’s of A stars
- 100-200 Myr old
- \( \sim \) exponential profile, \( r_0 \approx 0.1'' \) (0.4 pc)
- possible change for \( r < 0.03'' \).

STIS spectra 350-500 nm includes Ca II H and K, Balmer lines.
Kinematics of P3: razor-thin disc models
(Bender et al 2005)

P3 kinematics described well by simple exponential disc model,
\( i = 55^\circ \),
\( M_\bullet \approx 1.4 \times 10^8 \, M_\odot \).
Kinematics of P3: fat Schwarzschild models
(Bender et al 2005)

Best-fit model: thin disc around $M_\bullet = 1.4 \times 10^8 M_\odot$.
1-$\sigma$ range of thick-disc models is $(1.1 - 2.3) \times 10^8 M_\odot$. 
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1-$\sigma$ range of thick-disc models is $(1.1 - 2.3) \times 10^8 M_\odot$. 
2. The eccentric disc around the BH

(What’s happening outside P3?)
Background

Origin of P1–P2 eccentric disc: suggestions

- $m = 1$ instability in stellar disc (Jacobs & Sellwood, Bacon et al, ...)
- stellar remnant of eccentric gas disc that fed BH (Hopkins & Quataert 2010)

... 

Origin of P3: gas driven inwards P1–P2 potential, if $\Omega_p$ low enough? (Chang et al 2007).

![Diagram of orbits](image)
Origin of P1–P2 eccentric disc: suggestions

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- ...

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Motivation

“The laws of physics are perfect, but the human brain is not.”

What constraints can we hope to extract from observations?

- 3d shape, orientation
- internal orbit structure
- measurement of $M_\bullet$ (indep of P3).
WFPC photometry (Lauer et al 98):
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Data

OASIS fluxes (Bacon et al 2001):
Data

OASIS V (Bacon et al 2001):
OASIS $\sigma$ (Bacon et al 2001):
Data

STIS CaT long-slit kinematics (Bender et al 2005):
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More kinematics from, e.g.,

- van der Marel et al. (1994) (long slit)
- Kormendy & Bender (1999) (long slit, maj)
- Statler+99 (1999) (FOC)
2a. Three-dimensional, massless discs
Purely Keplerian potential: DF $f(a, e, l, \omega, \Omega)$. Assume disc has biaxial ($y, z$) symmetry.

Clump of orbits around $a = 1$, $e = 0.7$ projected along $z$: 
Purely Keplerian potential: $f(a, e, I, \omega, \Omega)$. Assume disc has biaxial ($y, z$) symmetry.

Clump of orbits around $a = 1, e = 0.7$ projected along $y$: 

![Image of clump of orbits](image-url)
Purely Keplerian potential: DF $f(a, e, l, \omega, \Omega)$. Assume disc has biaxial ($y, z$) symmetry.

Clump of orbits around $a = 1$, $e = 0.7$ projected along los:
Peiris & Tremaine (2003) took

\[ f(a, e, I) = g(a) \cdot e \exp \left[ -\frac{(e - e_m(a))^2}{2\sigma_e(a)^2} \right] \sin I \exp \left[ -\frac{l^2}{2\sigma_l(a)^2} \right]. \]

**Free parameters:**

- 3 \( g(a) \) radial sb profile
- +2 \( \sigma_l(a) \) thickness profile
- +5 \( e_m(a), \sigma_e \) eccentricity distn.
- +1 \( M_\star \)
- +3 \( (\theta_l, \theta_i, \theta_a) \) viewing angle
- =14 (neglecting centre)

**Fit:** WFPC \( V \)-band photometry and KB99 \( (V, \sigma) \) long slit.

**Predict:** KB99 LOSVD shapes; STIS, OASIS kinematics.
Some details:
For disc thickness:

\[ \sigma_I = \sigma^0_I \exp(-a/a_I). \]

For \( e_m(a) \), something of the form:

Motivation: encourage round P2, brighter than P1, with dip inbetween P1 and P2.
Models with $i = 55^\circ$ (right) are better fits than models aligned with the large-scale disc ($i = 77^\circ$, left).
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3d models with massless discs
(Peiris & Tremaine 2003: results)

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To avoid need to think about \(e_m(a)\), try instead
\[
f = \sum_i w_i \exp \left[ - \frac{(a - a_i)^2}{2\sigma_a^2} \right] e \exp \left[ - \frac{(e - e_i)^2}{2\sigma_e^2} \right] \sin I \exp \left[ - \frac{l^2}{2\sigma_{l,i}^2} \right].
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**Multiblob expansion**

Blobs centred on fixed pts in \((a, e)\) plane, plus
\[
\sigma_{l,i} = \{15^\circ, 30^\circ, 45^\circ\}.
\]

**Free parameters:**
- \(30 \times 9 \times 3 \ (n_a \times n_e \times n_i)\) blob weights;
  - \(\times 2\) if counter-rotating orbits included;
- \(M\);  
- orientation of disc on sky \((\theta_l, \theta_i, \theta_a)\).
Peiris & Tremaine (2003) had
\[ f = g(a) \exp \left[ - \frac{[e - e_m(a)]^2}{2\sigma_e(a)^2} \right] \sin \theta \exp \left[ - \frac{l^2}{2\sigma_l(a)^2} \right]. \]

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3d models with massless discs

Multiblob fit to WFPC
3d models with massless discs

Multiblob fit to OASIS fluxes
3d models with massless discs

Multiblob fit to OASIS V
3d models with massless discs

Multiblob fit to OASIS $\sigma$
3d models with massless discs

Multiblob fit to “STIS fluxes”
3d models with massless discs

Multiblob fit to STIS “V”
3d models with massless discs

Multiblob fit to STIS \( \sigma \)
Why is the fit to $\sigma$ so poor?

$V$ and $\sigma$ measured by fitting Gaussian model LOSVDs to spectra...

...my models assume $V$ and $\sigma$ are 1\textsuperscript{st} and 2\textsuperscript{nd} moments.

(not all LOSVDs agree this well...)
What does the disc look like? LOS projection:
3d models with massless discs
multiblob expansion results

What does the disc look like? Edge-on:
3d models with massless discs
multiblob expansion results

What does the disc look like? Face-on:
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Best-fit $M_\bullet \simeq 10^8 M_\odot$, $\theta_i \simeq 60^\circ$. DF looks like
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Dispersion of $I$ and $e$ as function of $a$:
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![Graph showing dispersion of $I$ and $e$ as function of $a$.]
Dispersion of $I$ and $e$ as function of $a$: \textit{NB: $\sigma_I \not\approx 0.5\sigma_e$!}
Summary of 3d models to date

- Either
  - provide only moderately good fit to photometry (PT03), or
  - barely fit kinematics (me)
- Don’t make use of LOSVD information
- Cavalier treatment of errors
- Neglect the mass of the disc ($\sim 0.1 M_\odot$)!

Three (semi-)independent implementations. All find $M_\bullet = 10^8 M_\odot$ to approximately 10%.

Degeneracies in fit: which $(a, e)$ features are essential? Dynamically informed prior on $(a, e, l)$ might be good.
2b. Two-dimensional, massive discs
T95: Massive disc makes orbits precess at different rates. Coherent eccentric disc won’t last long.

Assume BH-plus-disc system stationary in frame rotating at pattern speed $\Omega_p$.

Sridhar & Touma (1999): in nearly Keplerian potential, 
  - almost all orbits regular 
  - family of loop orbits that reinforce $l = 1$ perturbations

Problem (2d) 
What are $M_*$, $\Omega_p$ and $\rho_{\text{disc}}(x, y)$ for M31?
Weak 2d discs
(Statler 1999; Salow & Statler 2001, 2004)

SS04 assume DF

\[ f(a, e, \omega) = g(a) \exp \left[ - \frac{[e - e_0(a)]^2}{2\sigma_e^2} \right] \exp \left[ - \frac{\omega^2}{2\sigma_\omega^2} \right]. \]

Iterative scheme for finding \( \rho(x, y) \) given \( M_\bullet, \Omega \). E.g.,

Adjust free parameters to match photometry (along P1–P2 only) and kinematics.
Weak 2d discs
(Statler 1999; Salow & Statler 2001, 2004)

Best-fitting self-gravitating models in literature:
For 2d disc, we “know” \( \rho(x, y) \) from photometry:

Find combination of orbits that self-consistently reproduces this \( \rho(x, y) \) (with some assumed \( M_\bullet, \Omega_p \)).

Schwarzschild (1982): triaxial \( \rho(x, y, z) \) plus known \( \Omega_p \).

Present problem: \( \rho(x, y) \), unknown \( M_\bullet, \Omega_p \), obs errors
Schwarzschild’s method!
(Sambhus & Sridhar 2002)

Samples from orbit library, plus fit to photometry:
Kinematics weren’t fit (they assumed $M_\bullet = 3.3 \times 10^7 M_\odot$):
More up-to-date Schwarzschild models
(Calum Brown & JM, in prep)

Razor-thin model with $M_\bullet = 7 \times 10^7 M_\odot$, $M_{\text{disc}} = 2.1 \times 10^7 M_\odot$:
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Constraints on pattern speed:
More up-to-date Schwarzschild models
(Calum Brown & JM, in prep)

Constraints on BH mass:

\[ M_D = 2.1 \times 10^7 \, M_\odot \]
Two problems
Schwarzschild (1982): perfect knowledge of triaxial $\rho(x, y, z)$, orbit families
M31: $\rho(x, y, z)$ biaxial at best, simpler orbits, real data, fascinating system.

Summary of detailed modelling efforts

<table>
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Still no 3d models that include disc self gravity!
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Still no 3d models that include disc self gravity!
1 Black hole mass

2 Disk models
   - Data
   - 3d models with massless discs
   - 2d massive discs

3 summary