The Formation of Astronomical Images

Tod R. Lauer

Astronomical Images

- A set of points at which a quantity is measured within a space. Pictures, spectra, IFU “data cubes” are all examples of images.
- In this talk we will focus on 2-D flux maps with regularly spaced samples, but this is only a subset of possible images.
- To understand imaging, you need to understand how an object is represented by its image.

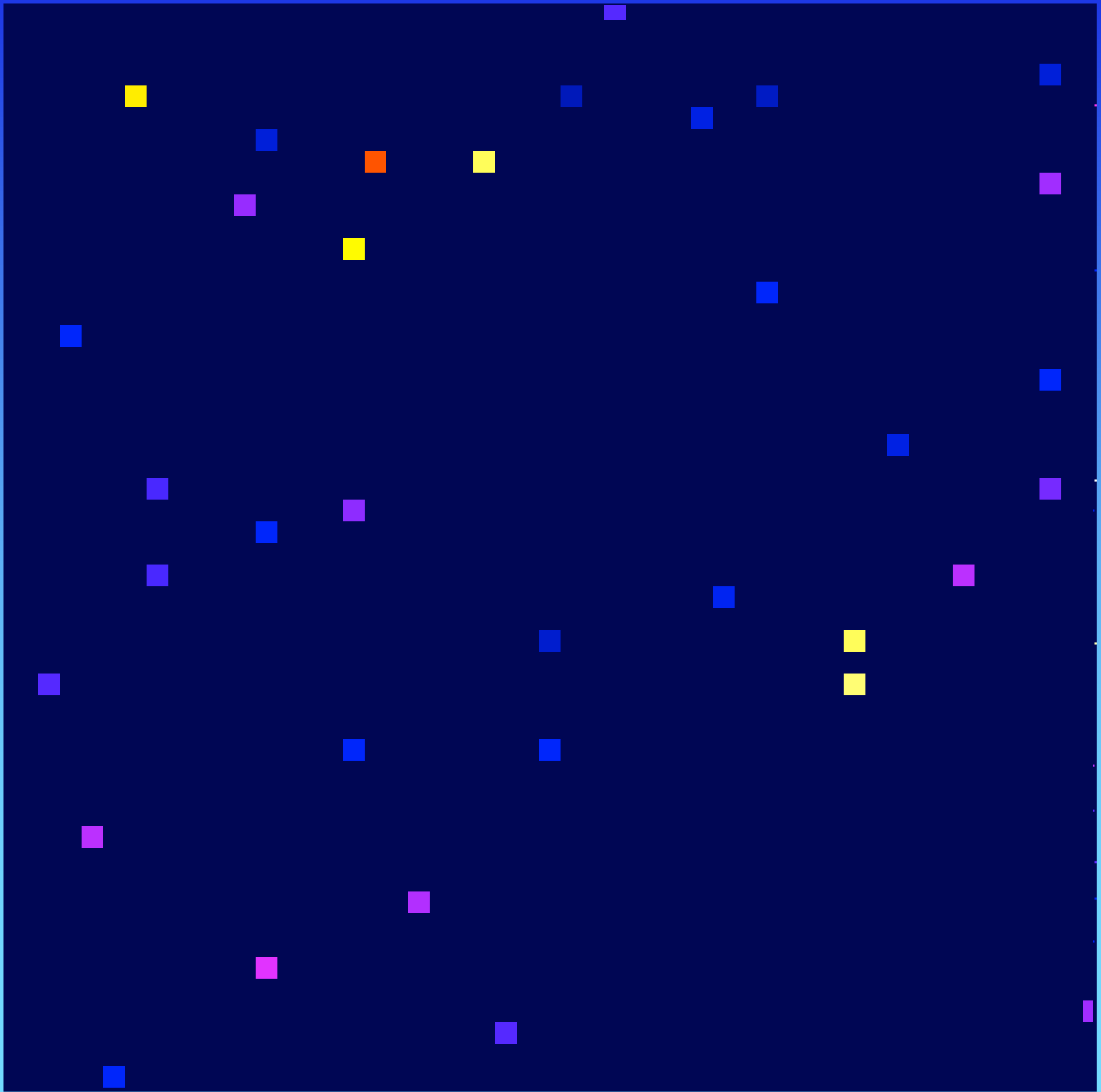
Image Formation

$$I = O \otimes (P \otimes \Pi) \times Ш + N$$

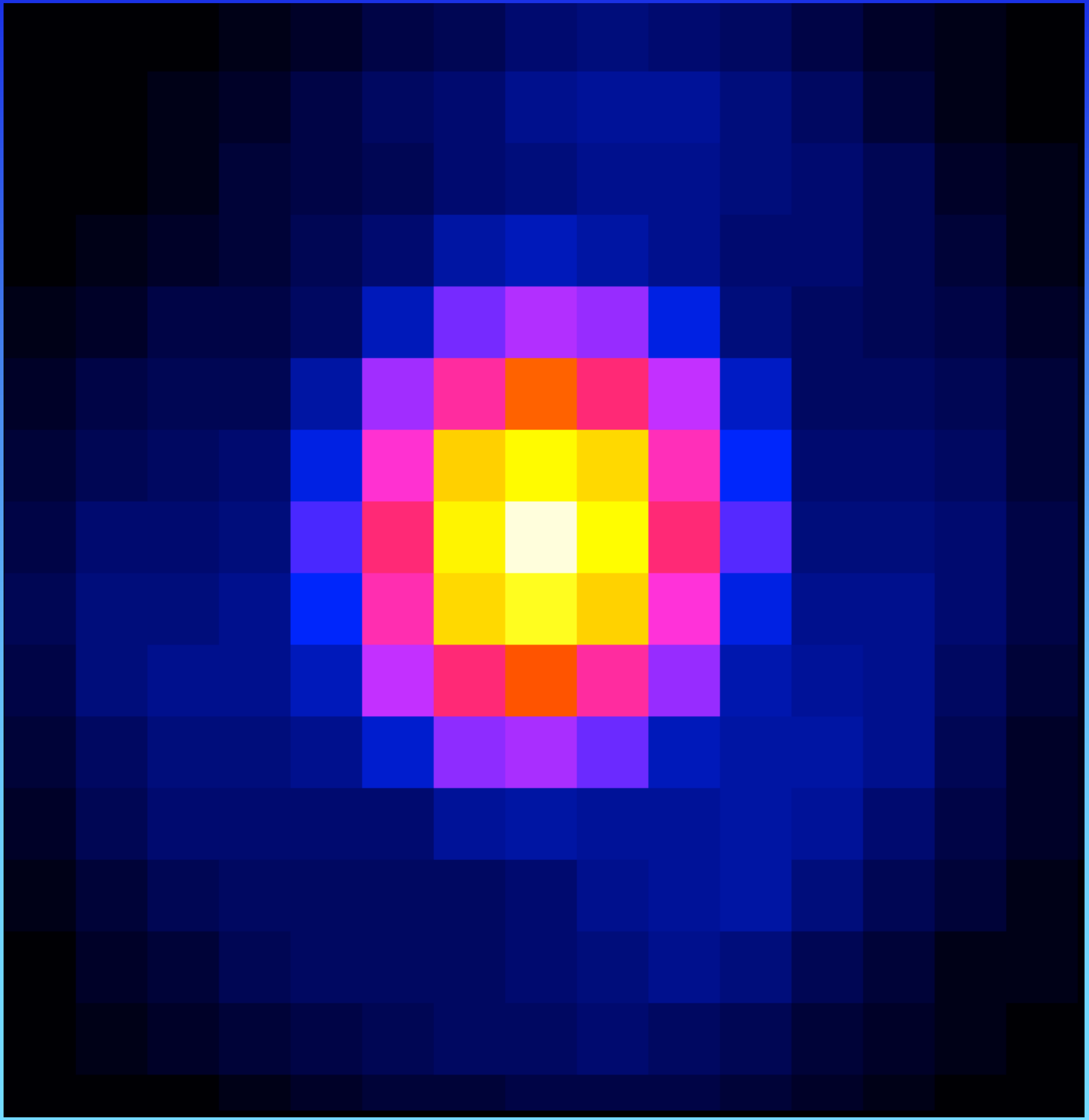
Effective or total PSF

I	Image Observed
O	Intrinsic Object
P	PSF
Π	Pixel Kernel
Ш	Sampling Comb
N	Noise

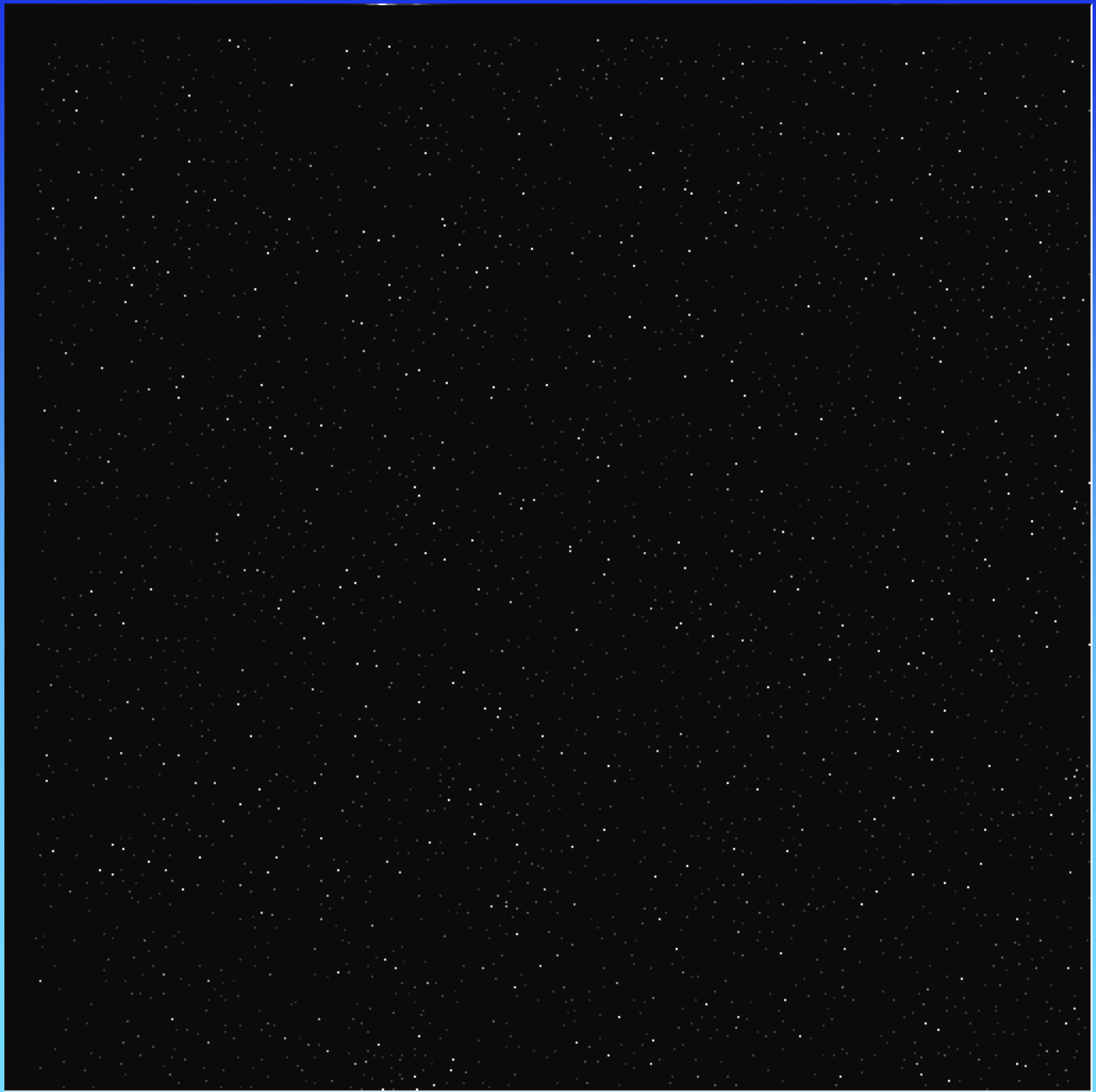
The Image Source



The Point Spread Function



The Observed Image

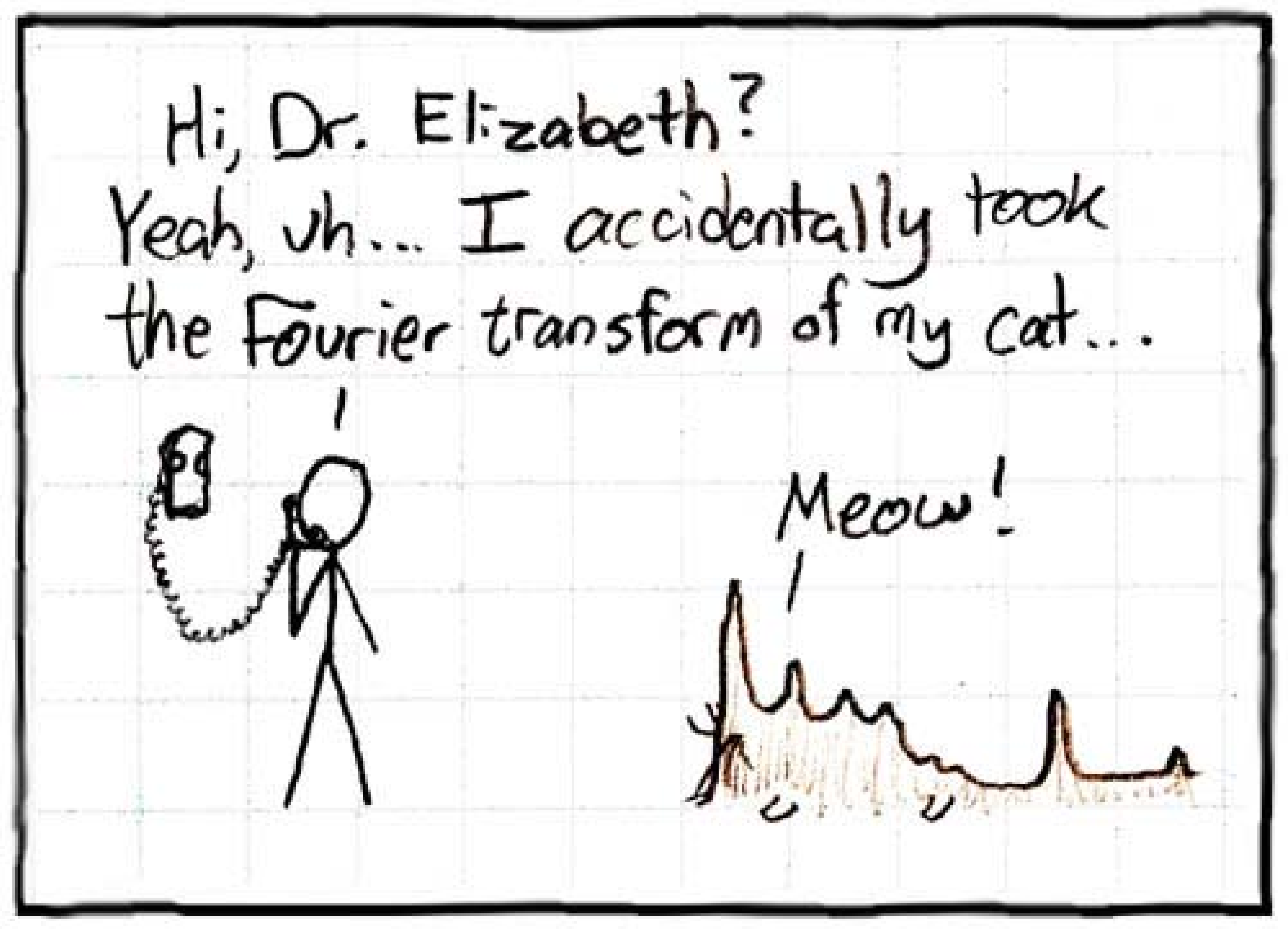


The Fourier Transform

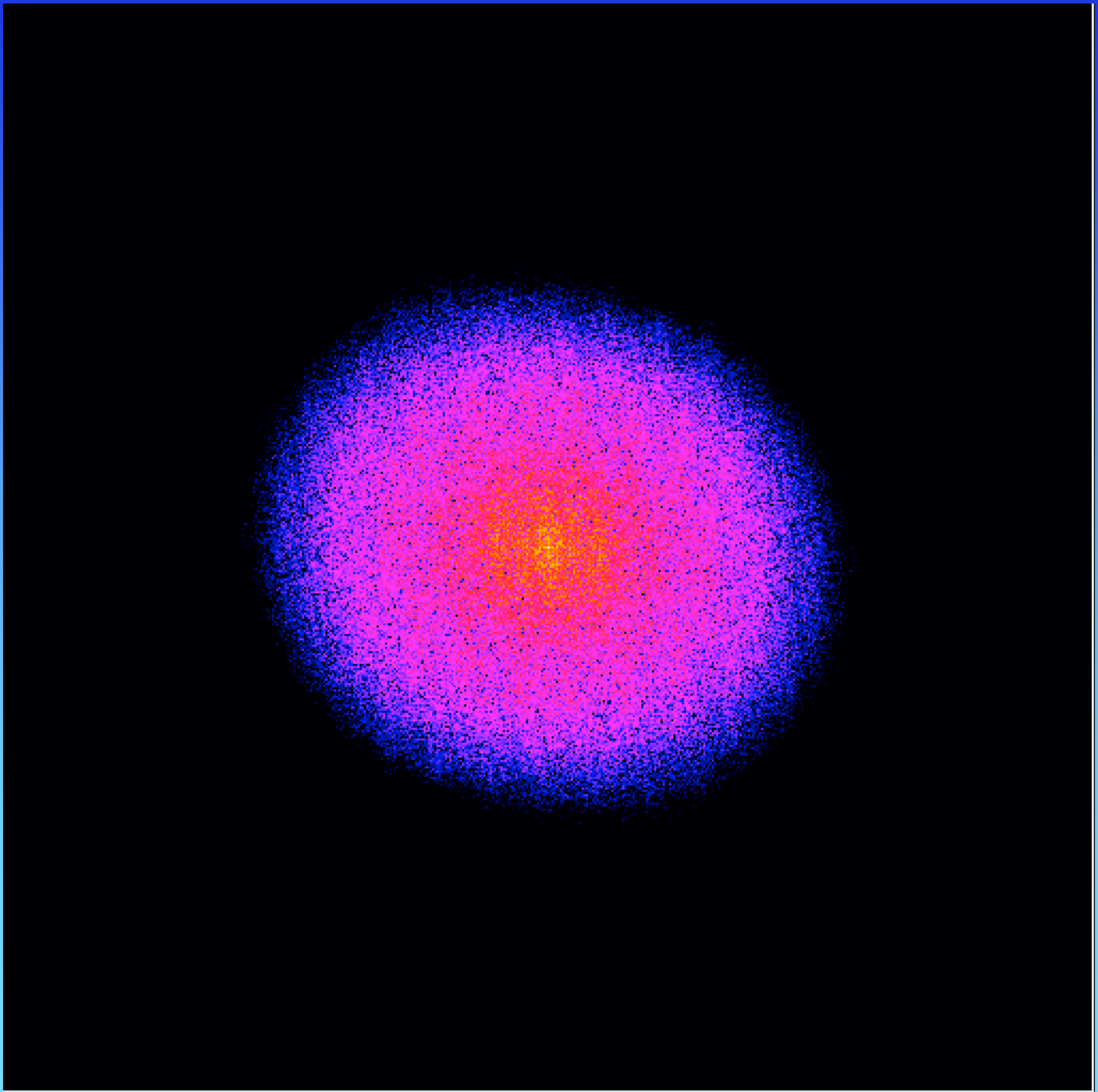
$$\overline{f(x,y)} = F(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)e^{-(2\pi ixu+2\pi iyv)}dx\ dy.$$

- The FT decomposes an image into a set of waves, with variable amplitude, phase, and direction.
- This allows information of different spatial scales to be isolated, analyzed, and processed.
- Convolution in Images space is multiplication in Fourier (and vice-versa).
- Complete symmetry and complementarity with Space.
- Reciprocal Fourier/Image space scale relationship.
- *Critical for understanding resolution, filtering, sampling, and on and on…*

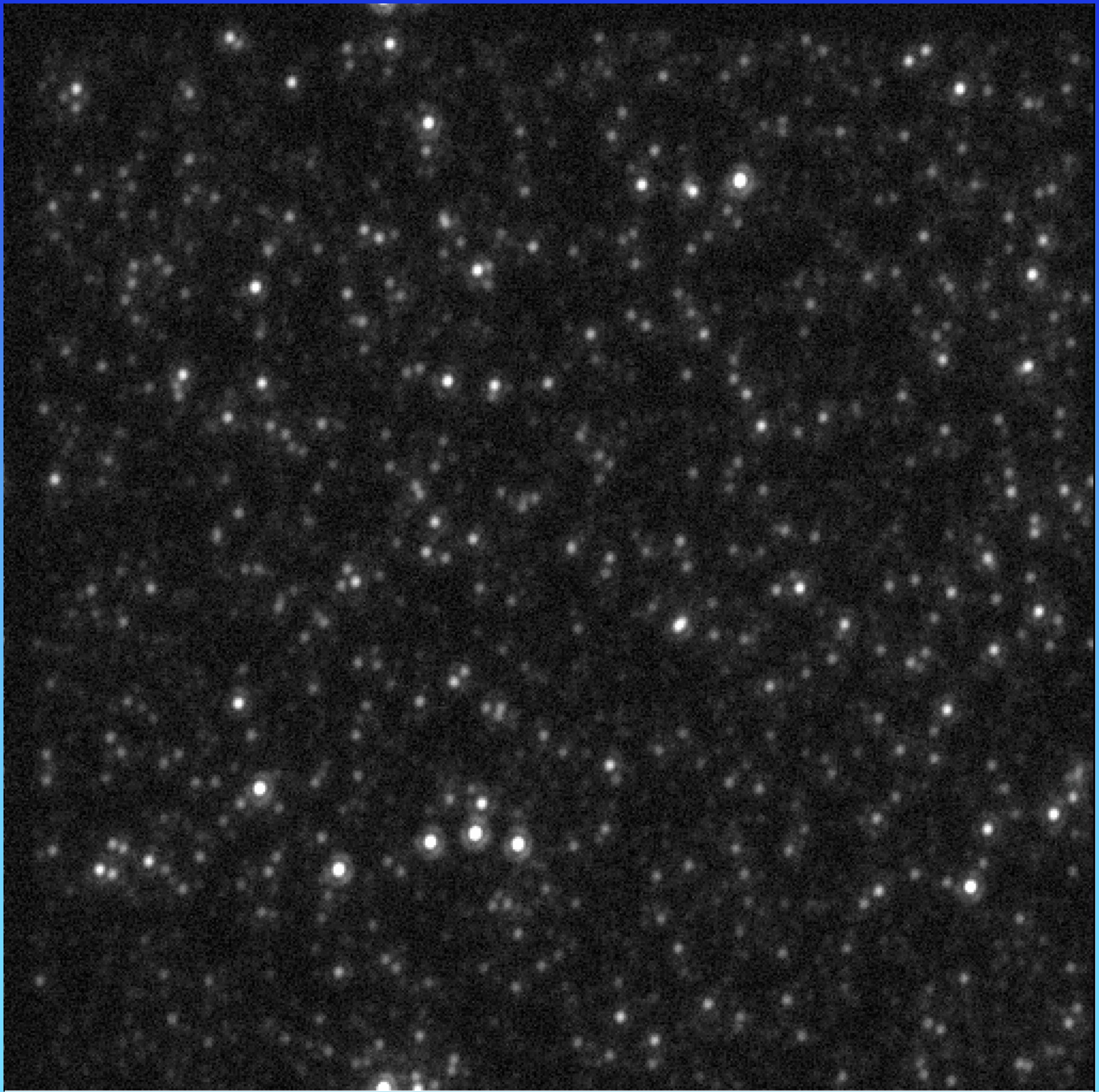
The Fourier Transform



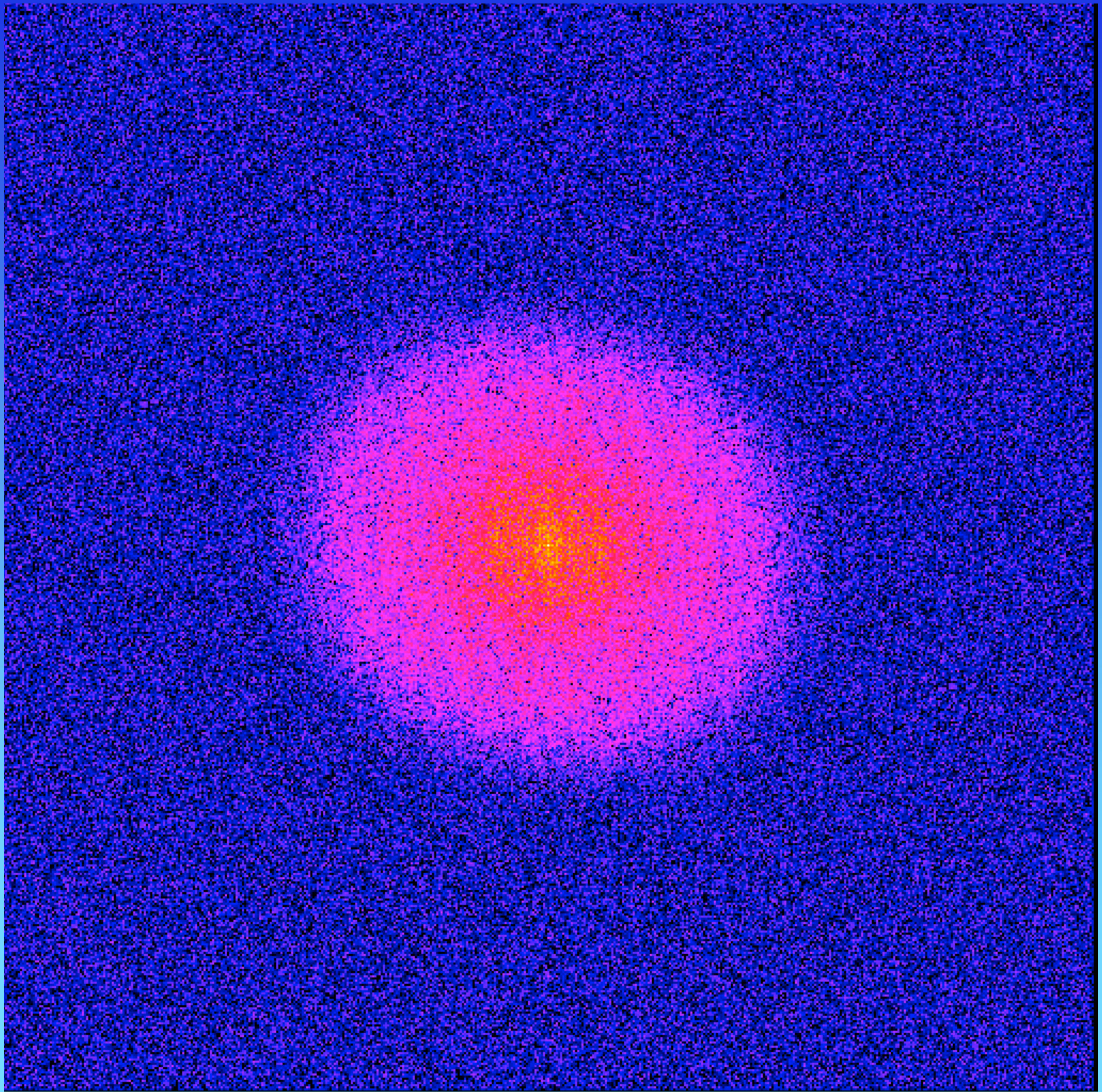
The Fourier Domain Perspective



And then add noise…



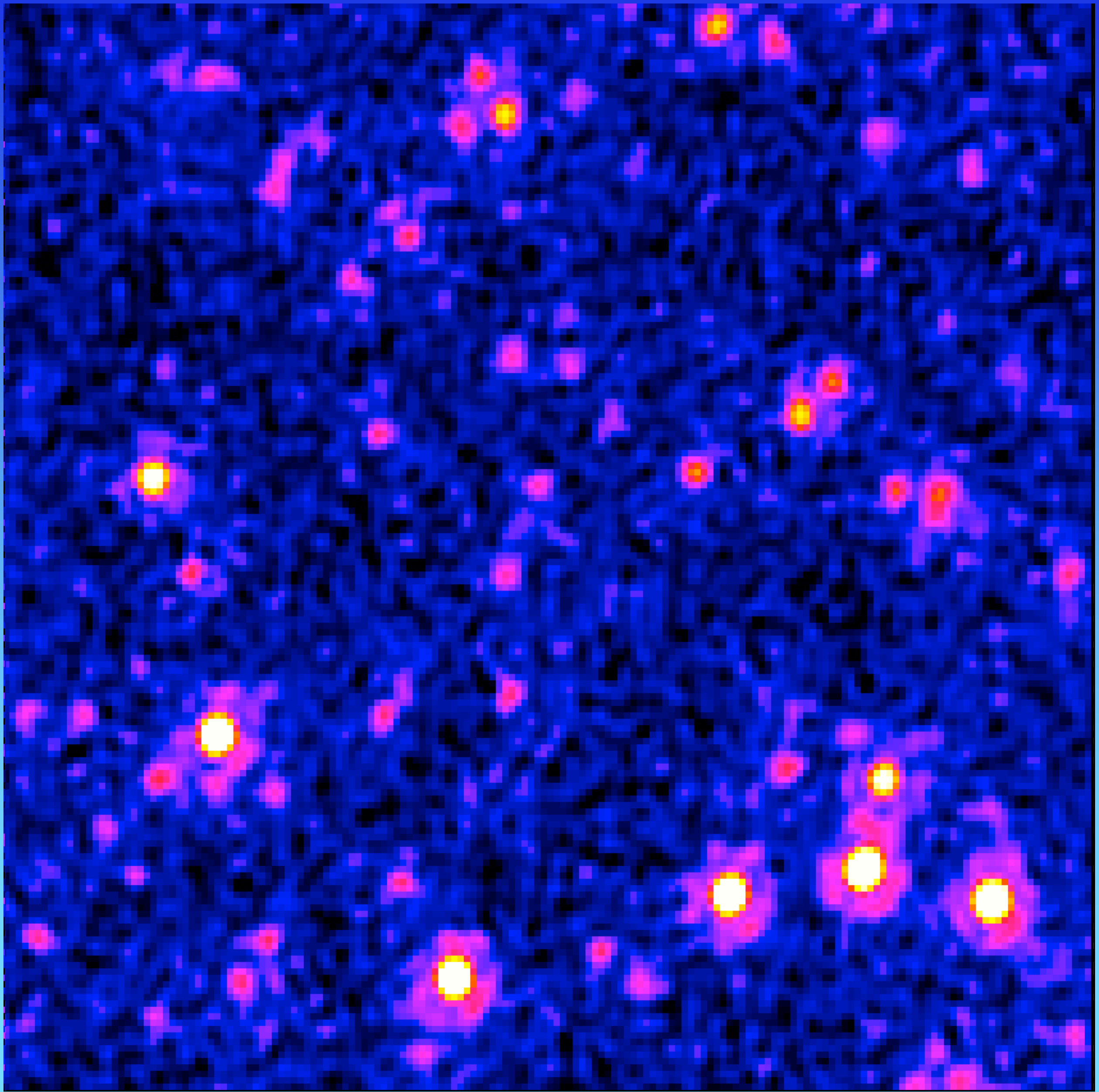
Noise in the Fourier Domain



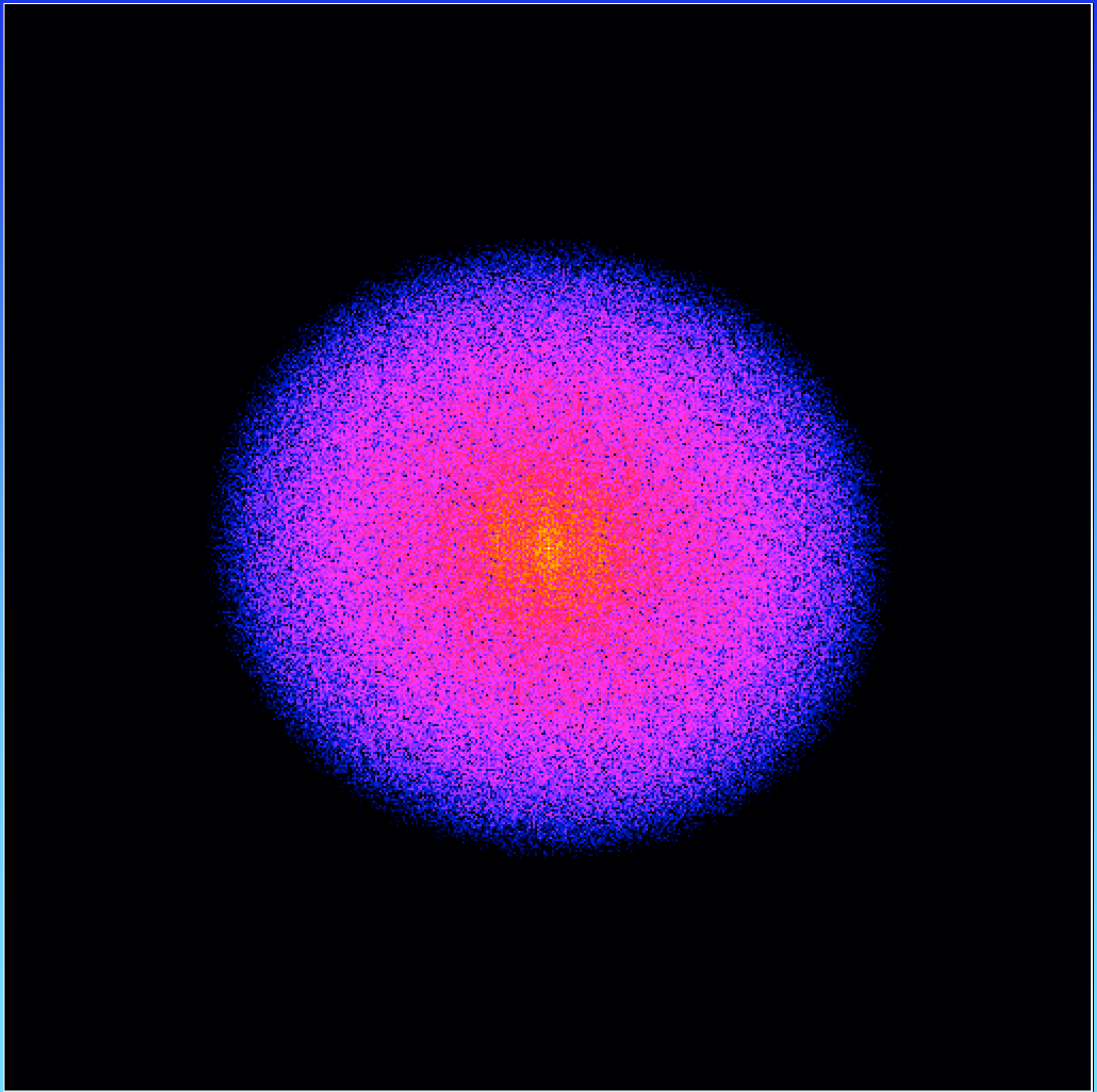
Understanding Noise

- Noise sets the limit on the photometric and structural information recovered from an image.
- Noise limits the spatial resolution of features. Noise can mimic fine-scale features.
- Separating noise from signal is the major task of image analysis.

Filtering Noise



Filtering in the Fourier Domain



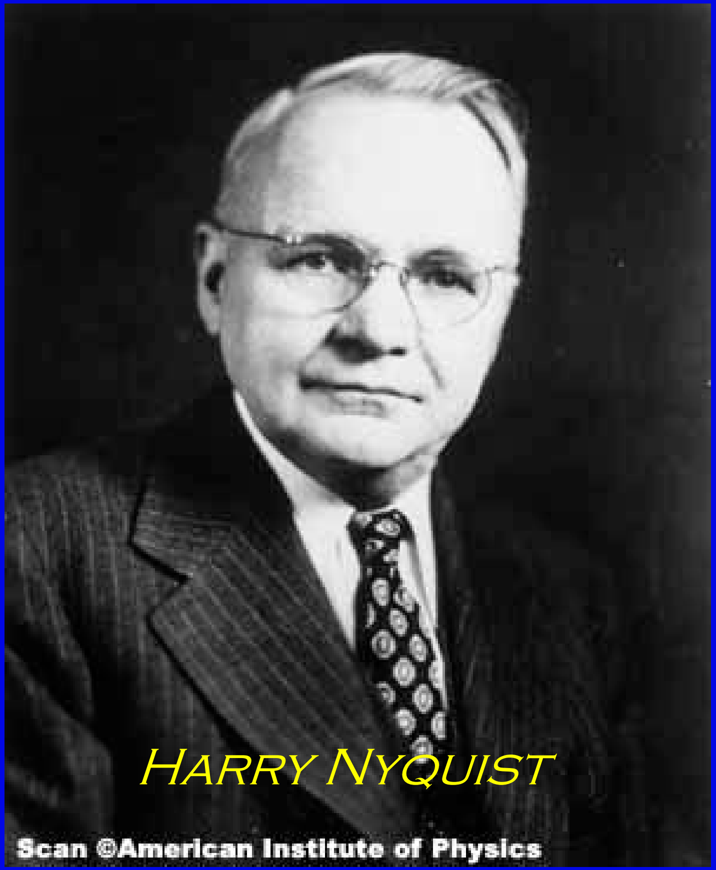
Filters and Measures

- Filters isolate signal from noise, specific signals from other signals, interpolate images, etc.
- All measurements from an image can be regarded as forms of filters.
- Good filtering means understanding exactly how both noise and signal respond to the filter.

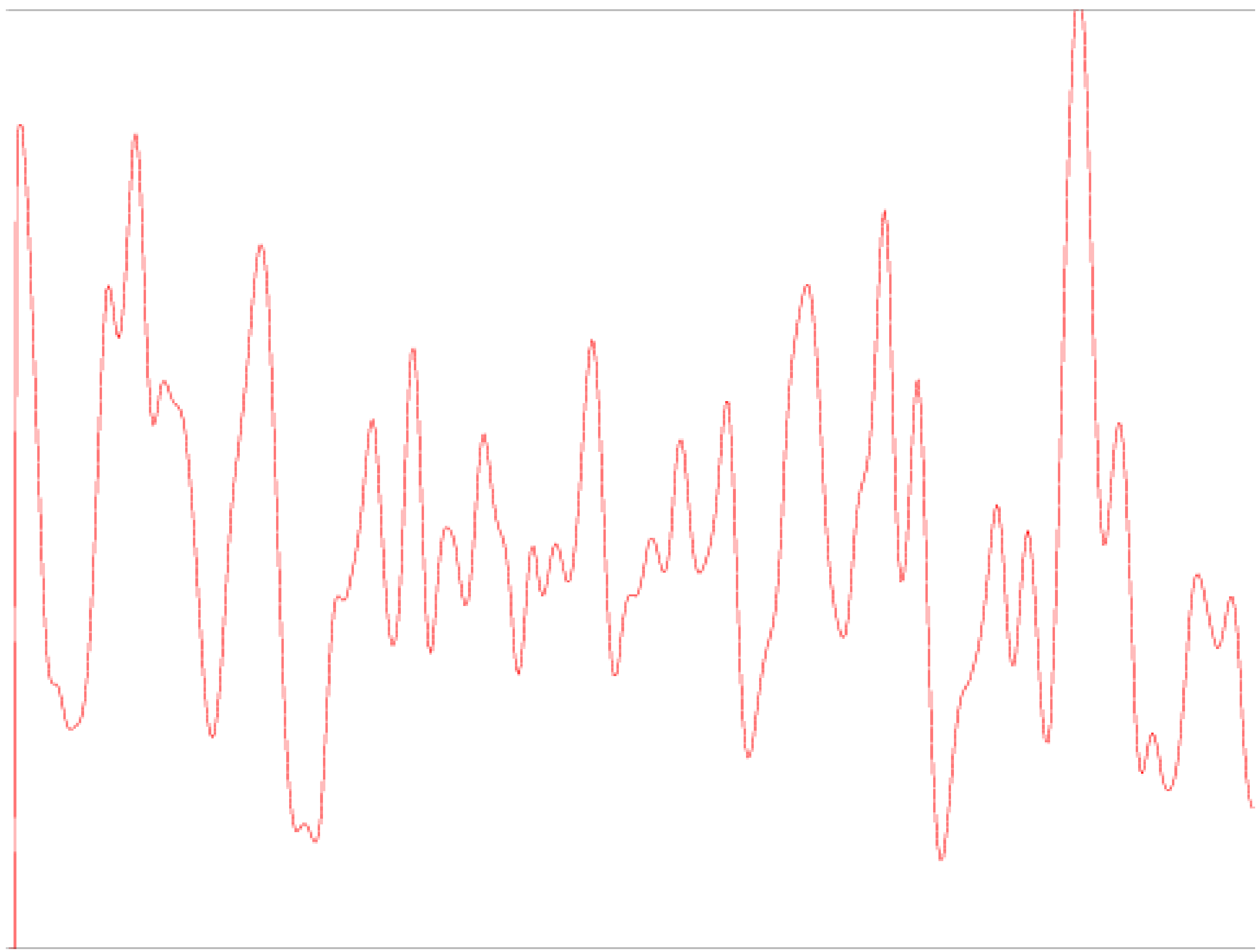
$$\mathrm{f}(\mathrm{x}_0) \equiv \int_{-\infty}^{\infty} \mathrm{F}(x_0 - x)\; x \; dx$$

Image Sampling

- Accuracy of photometry, astrometry, etc. depends on good sampling.
- Where possible, strive for "Nyquist-sampling," which requires sampling at 2× highest spatial frequency present in image.
- *Roughly-speaking - two pixels per PSF FWHM.*
- No need to oversample!



Sampling



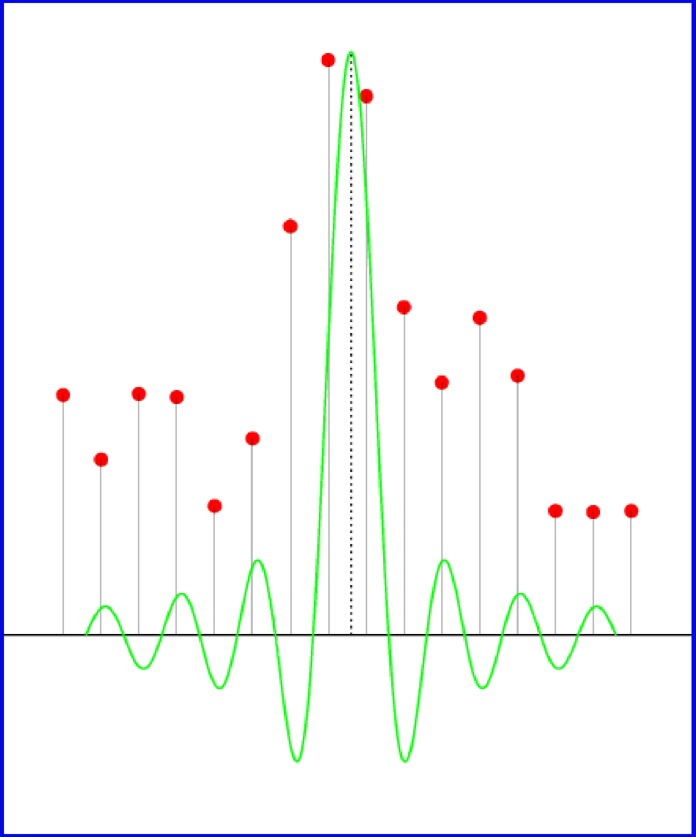
Sampling

- Sampling draws values at discrete points from a continuous function - *this includes the pixel kernel.*
- Samples = data pixels are pure delta functions. *Distinguish data pixels from detector and display pixels.*
- Under-sampling beats sampling frequency against spatial frequencies, aliasing them as bogus lower or higher spatial frequency power.
- Well-sampled data can be measured, interpolated, recast, etc. without resolution or photometric losses.
- Under-sampled data contain intrinsic photometric errors and cannot be resampled or interpolated without incurring additional signal degradation.

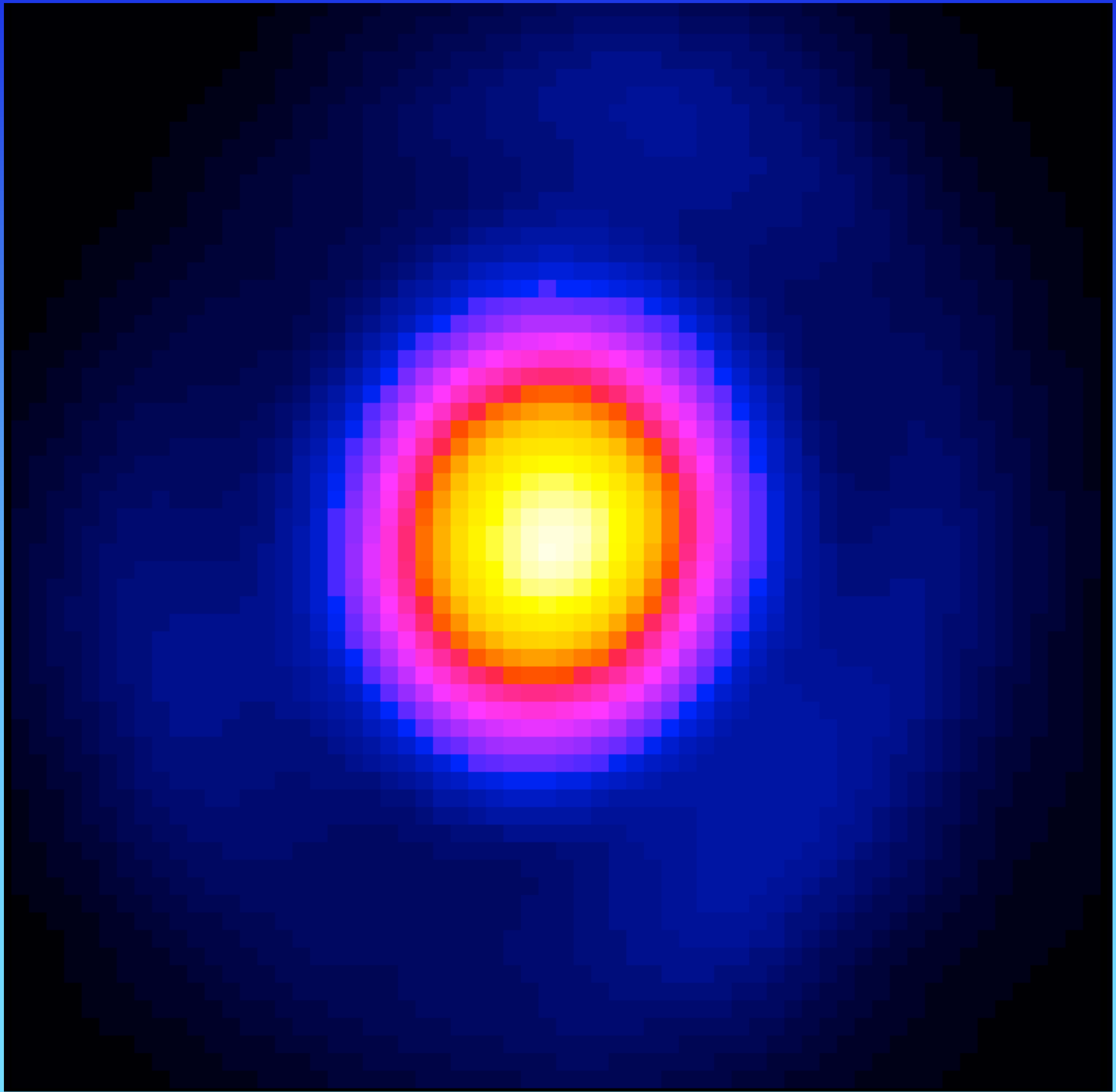
The Sinc, or Interpolating-Function

$$\mathrm{sinc}(\mathrm{x}) \equiv \frac{\sin \pi x}{\pi x}$$

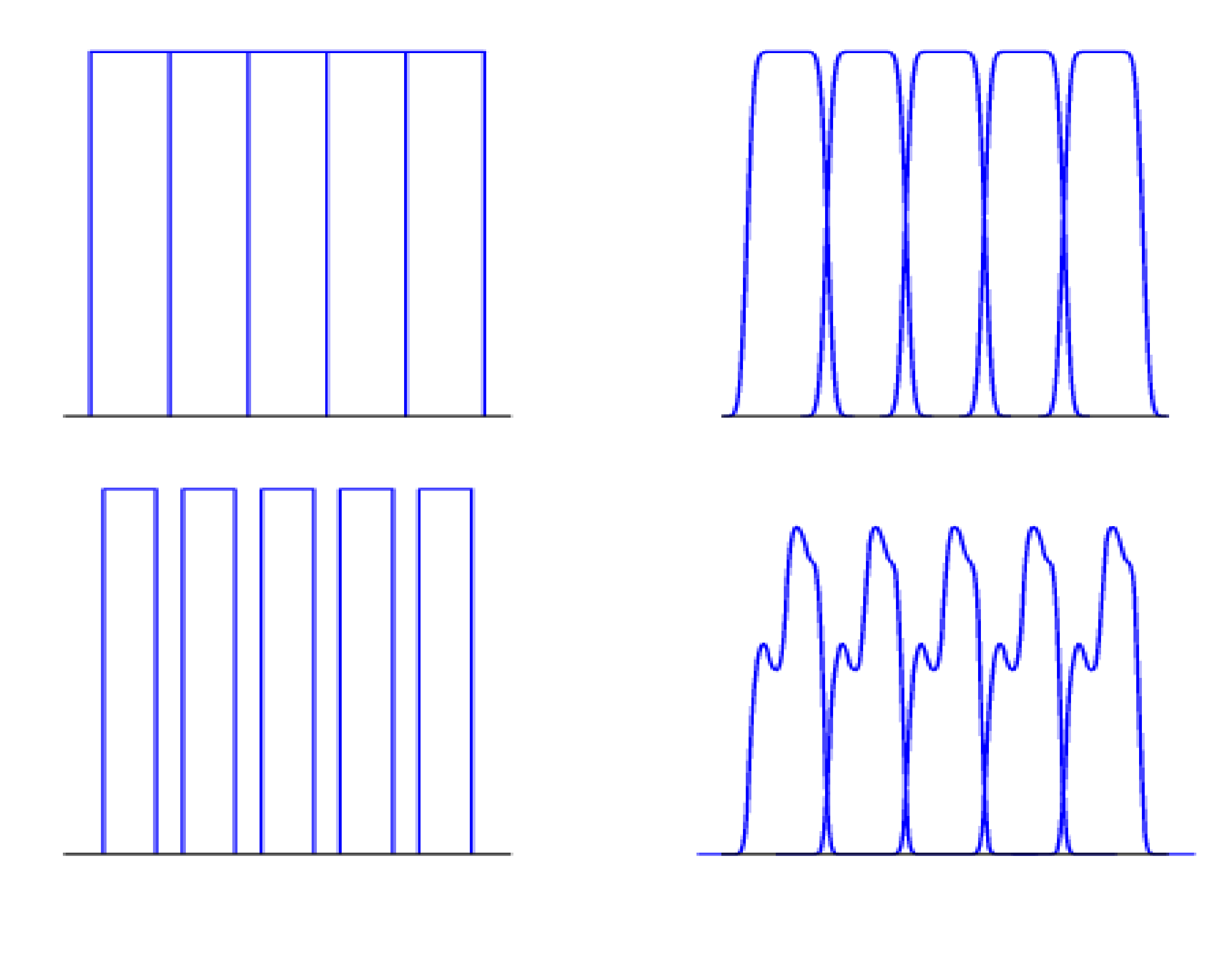
- The assumption that the image is well-sampled and continuous implies the use of *sinc(x). A "cutoff" box in the Fourier domain is sinc(x) in the image space.*
- *Sinc(x)* interpolates with no loss of resolution, smoothing, etc.
- *Any other interpolator is ad hoc.*
- Sinc is sensitive to artifacts, thus well-reduced images are required.
- *Sinc(x,y)* is separable - it results from multiplying to 1-D functions.
- Taper as needed.



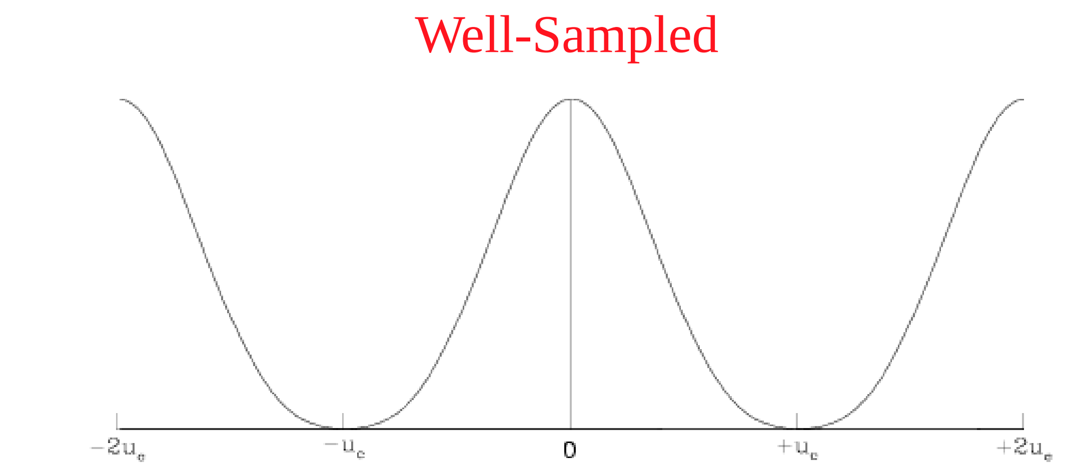
Display vs. Data Pixels

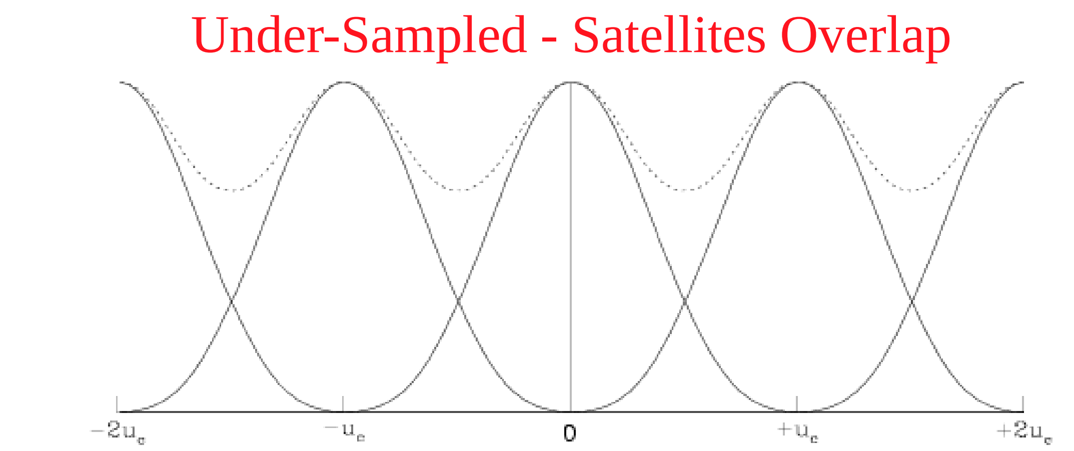


Detector Pixel Shapes



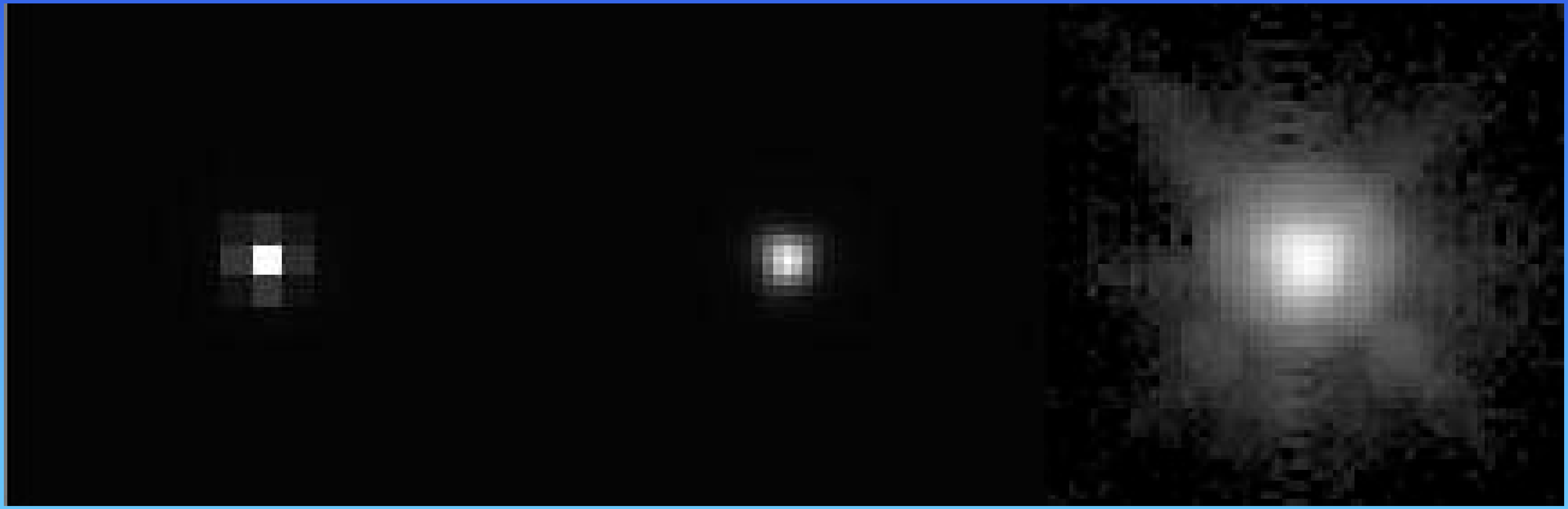
Aliasing in the Fourier Domain



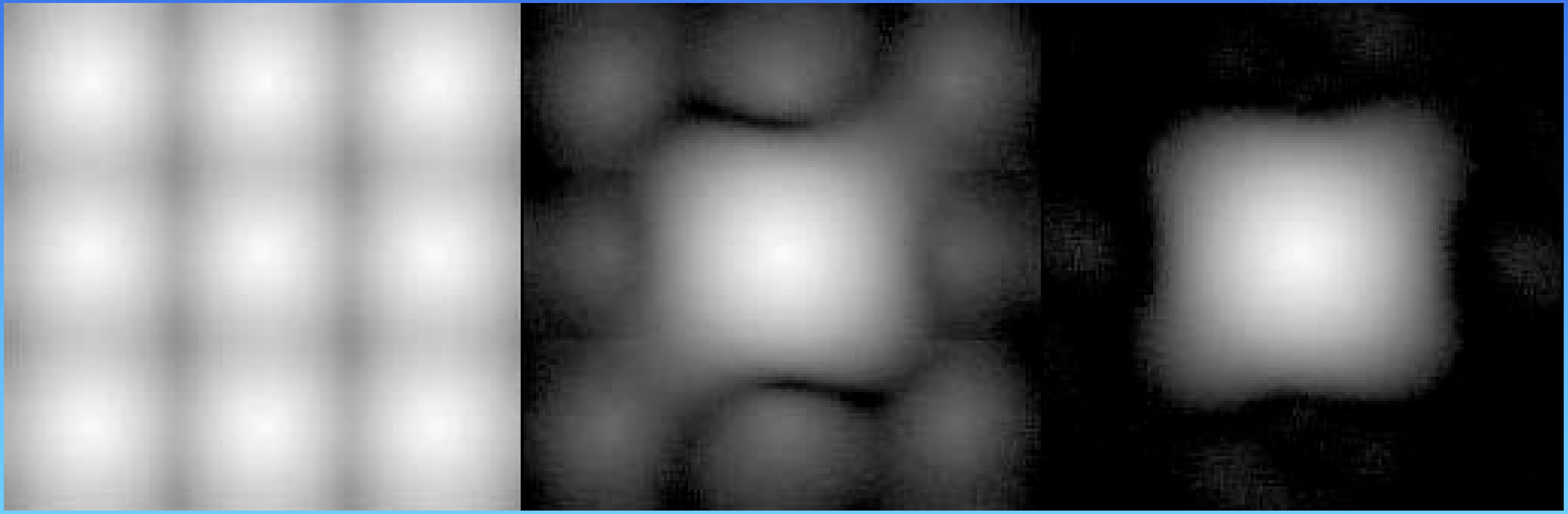


3X3 Sub-Sampled F555W WFC PSF

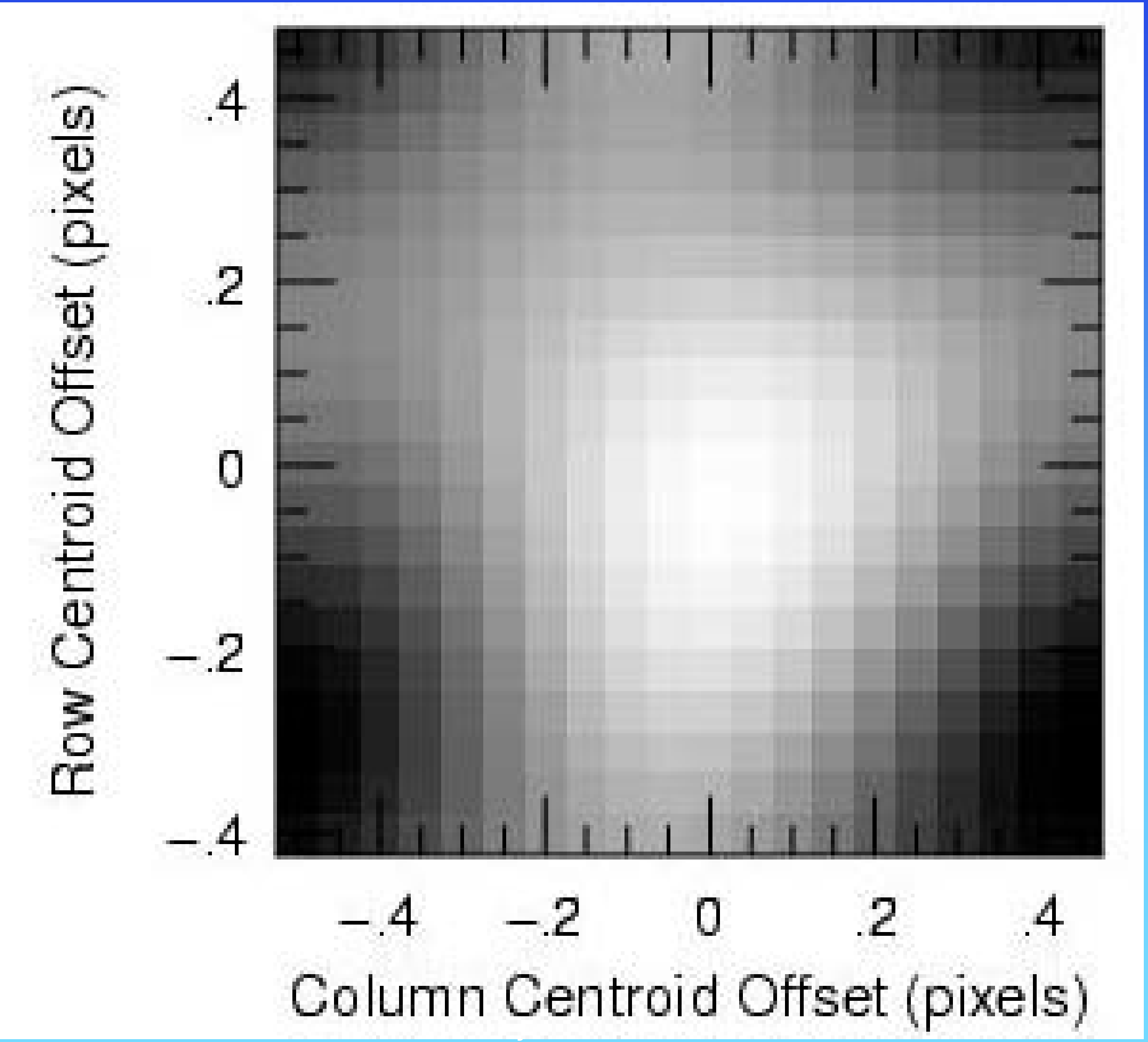
Single Native PSF



Fourier-Space 3X3 F555W WFC Image reconstruction



Photometric Errors Due to Undersampling



NIC3 J-Band

Dithering to Fix Undersampled Data

- Many cameras with large pixels produce undersampled data.
- The pixel + PSF sets the resolution.
- Shifting the camera by sub-pixel amounts recovers Nyquist-sampling given optical PSF + pixel kernal = total or effective PSF.
- Dithering now standard on HST and other spacecraft - does require PSF stability over dither sequence.
- Distinguish this dithering from large-scale dithering to mitigate detector defects and sensitivity variations.

Dithering