# DARK MATTER: LOCAL VOLUME DENSITY VERSUS TOTAL SURFACE DENSITY 

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#### Abstract

Spectrophotometric data have been obtained with the 2 dF spectrograph at Cerro Tololo Inter-American Observatory and with Hydra at the WIYN telescope for nearly 2000 A, F, and G stars toward the south Galactic pole. Using 1305 radial velocities, 2311 uvby $\mathrm{H} \beta$ photometric measurements, and 1621 Yale-San Juan SPM (Southern Proper Motion) absolute proper motions, peculiar velocities were derived to determine the galactic gravitational force $K(z)$ perpendicular to the Galactic plane, first described by Oort (published in 1932). Our results in local volume density, as derived from early-type stars (A0-F5) and giants, support Bahcall's $\rho=0.1 M_{\odot} \mathrm{pc}^{-3}$, implying that dark matter exists in the galactic disk. Our result for the total surface density derived using late-type stars ( G 1 and later) is equal to $34 M_{\odot} \mathrm{pc}^{-2}$, about $30 \%$ smaller than $46 \pm 9 M_{\odot} \mathrm{pc}^{-2}$ computed by Kuijken \& Gilmore, but greater than $23 M_{\odot} \mathrm{pc}^{-2}$ listed for the extended halo mass for $z<1.1 \mathrm{kpc}$ (Cox 1999). The derived behavior of $K(z)$ versus distance from the Galactic plane shows a well-known peak at about 400 pc . A second peak at about 1100 pc exhibited by dwarf main-sequence stars, but not for the giants, suggests that this second peak is likely due to a thick-disk population of metal-poor objects.


Key words: dust, extinction - Galaxy: disk - Galaxy: fundamental parameters - Galaxy: structure methods: statistical
On-line material: machine-readable table

## 1. INTRODUCTION

The method of star counts is a fundamental tool in the study of Galactic structure and evolution, providing us with a measurement of the density distribution of the galaxy's stellar components. When spectroscopic and astrometric data at the Galactic poles are used, the stellar population, chemical abundance, vertical density distribution, and the Galactic gravitational force perpendicular to the Galactic plane, $K(z)$, can be also analyzed using appropriate samples of stars. The problem of determining the local mass density of the galactic disk is usually tackled by measuring the density and velocity distribution of a population that traces the galactic potential as a function of distance " $z$ " from the plane.

The primary objectives of this study are to investigate the velocity distribution, to search for evidence of a thick disk, and to conduct a dynamical study of dark matter in the

[^0]Galactic disk. The relationship between observed density and velocity distributions of stars depends upon the gravitational field and whether the system is in dynamical equilibrium. The difficulties of selecting tracer stars for a $K(z)$ study were outlined by King (1989).

In an earlier study, Oort (1932) used F stars to first suggest the existence of " missing mass" in the Galaxy. Bahcall (1984a, 1984b) used A and early F stars and giants to compare with his galaxy models, suggesting that about $50 \%$ of the mass in the Galactic disk is in the form of dark matter. At the end of the 1980s, using G and K dwarfs, Kuijken \& Gilmore (1989) found that there is no robust evidence for dark matter in the disk. For a review of the early studies, see Philip \& Lu (1989). Other recent discussions can be found by Freeman (1992), Fuchs \& Jahreiss (1998), Gould, Popowski, \& Terndrup (1998), Norris (1998), and Chen (2000).

## 2. THE SURVEY

A catalog of positions and identifications for $3178 \mathrm{~A}, \mathrm{~F}$, and G stars in a $40 \mathrm{deg}^{2}$ area centered at the south Galactic pole (SGP) have been reported by Lu, Miller, \& Platt (1992, hereafter referred as Paper I). One UK Schmidt Telescope Unit (UKSTU) film (including SA 141) was used as a guide for this study. The film, originally obtained to search for


FIg. 1.-Heavy line: UK Schmidt, five plates cover $12^{\circ} .6 \times 12^{\circ} .6$, centers are marked as a large hatched circle; light line: Curtis Schmidt, nine plates cover $12^{\circ} \times 12^{\circ}$, centers are marked as small hatched circles; hatched area: Bok and Basinski's Photographic survey (Bok \& Basinski 1964) using $B-V$, five plates covering $1.5 \times 1.5$ each, centers are marked as open circles.
quasars, is overexposed for stars brighter than magnitude $V \sim 13$. An additional set of nine $5^{\circ} \times 5^{\circ}$ Michigan Curtis Schmidt plates were taken at Cerro Tololo Inter-American Observatory (CTIO; see Fig. 1). Details of the observations and data analyses of these series have been reported by Lu and his coworkers (see references). The tracer group in this study was selected using thin prism spectral plates obtained at CTIO. This thin prism yields a dispersion of $1360 \AA$ $\mathrm{mm}^{-1}$ to a limiting magnitude of $V \sim 16.5$, reaching to about 4 kpc for early main-sequence $F$ stars and giant stars and to about 1 kpc for late main-sequence $G$ stars. The selection was based on a visual inspection of the thin-prism plates and the examination of spectral features in Yale PDS digital scans of those plates using the IRAF surface plot package (see Figs. 2 and 3, Lu 1991).

## 3. OBSERVATION AND DATA

Spectrophotometric observations for this project were started in 1988 and obtained on 58 nights at CTIO and two nights at KPNO. All photometric data were obtained at CTIO using the ASCAP (Automated Single Channel Aperture Photometry) on the Yale 1 m telescope while the 0.9 m was exclusively used for the CCD with the same standard set of TI chip No. 3 and $u v b y \mathrm{H} \beta$ filters set No. 2 throughout the entire study. Discussions of the spectrophotometric calibrations were reported elsewhere, as noted in the references. A very brief summary is presented here. All calibrations were done at CTIO under the supervision of the visitor support staff using strictly controlled procedures. Flat-field corrections were carried out using an average of five frames each of dome and sky flats. An average of 25 bias frames were taken with a mean of $2-3 e^{-} \mathrm{ADU}^{-1} \mathrm{rms}$, and a dark frame was obtained with a total of $25,000 \mathrm{~s}$ integration.

All these frames were systematically obtained according to the procedures outlined in the CTIO Facilities Manual (CTIO 1990, § V, p. 16, or see the NOAO Web site) ${ }^{3}$ at the beginning of each run. In general, 10,000 counts for each flat field were obtained, however, due to the throughput differences of the different filters, the number of seconds required to achieve 10,000 counts varies from a 10 s integration for $y$ to 1200 s for $u$. Therefore, the statistical errors for each flat field, in $u, v, b, y$, and $\mathrm{H} \beta$ is about $2 \%$.

Two TI chips were used throughout, a $512 \times 512$ TI chip (chip No. 2) was used in the earlier stage of observation, a $2 \mathrm{~K} \times 2 \mathrm{~K}$ TI chip (chip No. 3) was used after 1994. Since the 0.9 m of CTIO was exclusively devoted to photometry, cold box and setup were undisturbed unless due to telescope maintenance. The characteristics of the CCD and filters which were detailed documented in CTIO handbook will not be discussed here (see CTIO 1990, Appendix A).

### 3.1. Spectroscopy

Spectra have been observed using the 2 dF spectrograph at the CTIO Yale 1 m telescope with a dispersion of $43 \AA$ $\mathrm{mm}^{-1}$, and with Hydra using the 3.5 m WIYN telescope at a dispersion of $22 \mathrm{~A} \mathrm{~mm}^{-1}$. The details of the observation, reduction, and radial velocity accuracy have been discussed in $\mathrm{Lu}(1991,1993)$. Since it is not possible to distinguish giants from dwarfs using objective-prism spectra, MK spectral classification standards have been used (Keenan 1963). The distribution of spectral and luminosity types was first shown in Table 2 of Paper I. The distribution of spectral types for stars earlier than F5 and the spectral comparison between the 2 dF spectral types and those of Slettebak \& Brundage (1971) are shown in Figs. 5, 7, and 8 (Lu 1994).

The average spectral type is around F9.5. The mean radial velocity of 1297 stars determined from 1309 spectra is $7.19 \pm 27.13 \mathrm{~km} \mathrm{~s}^{-1}$ (s.e.) (Lu 1993; see Fig. 2).

[^1]

Fig. 2.-Histogram of radial velocities in the SGP region. The average radial velocity is $\left\langle R_{V}\right\rangle=7.19 \pm 27.13 \mathrm{~km} \mathrm{~s}^{-1}$ (unit weight rms error).

### 3.2. Direct CCD and ASCAP Photometry

As mentioned above, the 0.9 m telescope at CTIO was used exclusively for the CCD photometry. Direct CCD images with a field of view (FOV) of $512 \times 512$ pixels or $5^{\prime} \times 5^{\prime}$ were acquired before 1994 . A $2 \mathrm{~K} \times 2 \mathrm{~K}$ CCD chip was employed after 1994, with a FOV of $13.5 \times 13.5$. ASCAP measurements were done solely with the Yale 1 m telescope at CTIO. The detailed specifications of the CCD and ASCAP employed can be found in the CTIO Facilities Manual (CTIO 1990).

Photometric standards and magnitude transformations have been described by Lu (1989) and Lu et al. (1992). A total of 2311 stars in the $u v b y \mathrm{H} \beta$ system were observed using the two telescopes. The observations show that the ASCAP system is far superior for single object photometry due to the very low quantum efficiency at the $u$ band of the CCD chip. The system is designed for single object photometry on a noncrowded regions project and is generally more suitable and efficient than CCD imaging using the Strömgren system, since the $u$ bandpass in a photon counting system is about a factor of 3 faster than in the CCD system for comparable accuracy. There appear to be no systematic differences between the CCD and ASCAP photometric data (Lu 1989; Lu et al. 1992).

A total of 4554 photometric observations were made and $18 \%$ of the stars have multiple observations. Figure 3 shows the distribution of the magnitude errors. Large scatter on some bright stars may be caused by bad pixels or columns near the target stars that could not be avoided or to poor photometry accuracy close to the edge of the CCD frame. Those data were given lower weight in the final analyses.

Furthermore, spectroscopic survey has been completed only to about 14.5 mag in $y$, as mentioned in the following section, thus, without radial velocity, all these faints objects would not have been included in the final $K(z)$ analysis.

The mean color index for the 2311 stars is $\langle b-y\rangle=$ $0.359 \pm 0.080 \mathrm{rms}$, a value typical for a F9 V star. Objects in common with the photoelectric observations by Eriksson (1978) were used as a comparison (Fig. 4). A slight bend occurs in the diagram of $b-y$ versus $B-V$, and it is due to


Fig. 3.-Plot of $y$ magnitude vs. its error for 4554 observations


FIG. 4.-Strömgren $y$ vs. $V$ and $b-y$ vs. $B-V$, where the $U B V$ data come from the photoelectric observations by Eriksson (1978).
the difference of color systems for the stars of different types. We also carried out a comparison with the $u v b y \mathrm{H} \beta$ catalog compiled by Hauck \& Mermilliod (1998). Dozens of common bright stars were inspected in four colors, $b-y, m_{1}, c_{1}$, and $\mathrm{H} \beta$ (see Fig. 5). Good linearity was found in $b-y, c_{1}$, and $\mathrm{H} \beta$, but a loosely scattered distribution occurred in $m_{1}$. Most of the bright stars have multiple observations with high signal-to-noise ratio and served as secondary standards with more than 10 observations each in our survey. Therefore, the discrepancy shown in $m_{1}$ comes perhaps from the systematic differences among the various photometry sources of Hauck \& Mermilliod (1998).

After 1994, a total of $10 \mathrm{deg}^{2}$ area within the $40 \mathrm{deg}^{2}$ survey field was observed with the $2 \mathrm{~K} \times 2 \mathrm{~K}$ chip. All stars in this $10 \mathrm{deg}^{2}$ field were photometrically selected using the IRAF DAOPHOT task to the limiting magnitude of 17 in $y$ magnitude, regardless of their spectral types and color. There are a total of 1116 additional stars with magnitude


Fig. 5.-Color comparisons between this study (CLMvA) and Hauck \& Mermilliod's (1998) catalog
brighter than 17, and most of them are red objects. At present, the survey is complete to $V \sim 14.5$ in both spectra and photometry. Since, the $2 \mathrm{~K} \times 2 \mathrm{~K}$ CCD fields reach much fainter objects than the spectroscopic data, this portion of the sample became important to test the completeness of the selection of F and G stars (Fig. 6).
In general, the atmospheric extinction at CTIO is very stable throughout the years, hence, a standard set of values ( $b-y=0.045, m_{1}=0.06, c_{1}=0.05, y=0.17$, and $\mathrm{H} \beta=$ 0.14 ) was used, which is slightly different from the CTIO user's manual. However, after the 1992 volcanic eruption of Pinatubo in the Philippines, the mean coefficients for six nights in 1992 and in 1993 are 0.748 for both $y$ and $\mathrm{H} \beta$ compared with the normal value of 0.17 or smaller.

### 3.3. Reddening Determination

The reddening is derived using the reddening free parameter of 2 dF spectral types and $\mathrm{H} \beta$ index. The correlation between these two parameters indicated a linear relation between $\mathrm{H} \beta$ and the 2 dF spectral types. The relationship
between the observed 2 dF spectral type and the observed reddened $b-y$ color provides the observed mean $\langle b-y\rangle_{\text {SpT }}$. Using unreddened $\mathrm{H} \beta$ versus reddened $b-y$ would provide a second set of $\langle\langle b-y\rangle\rangle_{\mathrm{H} \beta}$. The difference between $\langle b-y\rangle_{\mathrm{SpT}}$ and $\langle\langle b-y\rangle\rangle_{\mathrm{H} \beta}$ yields the reddening. This procedure yielded a reddening value of 0.018 for $\mathrm{F} 0,0.020$ for G 0 , and 0.016 for K 0 stars, while the reddening value $E(B-V)$ derived from the Burstein \& Heiles (1982) map for SGP is 0.014 . The peak value of $b-y$ of this survey is centered around F9.5, or $\langle b-y\rangle \simeq 0.345$, therefore, a reddening value of 0.018 is adopted.

### 3.4. Standards and Secondary Standards

Photometric standards from Crawford (1975) were used in the 1988 runs. Neutral density filters of $2.5-7.5$ mag used for Crawford's bright standards caused some calibration problems because the transmission curves of the combined neutral density filter set is not well defined as reported earlier ( Lu 1993). In addition, a systematic difference appears to exist in Twarog's secondary standards (Twarog 1984).


Fig. 6.-Completeness comparison for the spectrophotometric observations. Plus signs are for preselected $F$ and $G$ early stars, circles for $F$ and G stars with spectra, crosses for $2 \mathrm{~K} \times 2 \mathrm{~K}$ CCD measurements of $b-y<0.430$ (G5), while the heavy lines represent the completeness limits.

After 1989, a new set of HD stars fainter than seventh magnitude was selected from the data of Olsen (1983) and McFadzean, Hilditch, \& Hill (1983), which were used as secondary standards within the $40 \mathrm{deg}^{2}$ of the SGP. Results for these SGP secondary standards were reported earlier by Lu (1993). The number of observations (col. [13] in Table 1) for the standard stars is normally larger than six, and it can range up to 30 .

### 3.5. Spectrophotometric Data

In summary, there are 1309 spectra, 1297 radial velocity measurements, 2615 photometric measurements, 1632 stars with absolute proper motions, and 1868 stars in common with the Guide Star Catalog.

Spectroscopic and photometric parallaxes were adopted using the following calibrations:

1. For spectroscopic parallaxes, Keenan's spectral luminosity relation was used (Keenan 1963, p. 92—Table 6).
2. The photometric parallaxes for the dwarf mainsequence stars were derived using Laird, Carney, \& Latham (1988), where

$$
\begin{equation*}
M_{\text {phot }}(V)=10.11(b-y)+0.68 \tag{1}
\end{equation*}
$$

3. Photometric parallaxes for giants were determined by Lu et al. (2000) in the following form

$$
\begin{equation*}
M_{\text {phot }}(\mathrm{III})=-1.305(b-y)+1.406 \tag{2}
\end{equation*}
$$

4. The spectral type versus $b-y$ color transformation derived was

$$
\begin{equation*}
\mathrm{SpT}=28.92+31.78(b-y)+71.06(b-y)^{2} \tag{3}
\end{equation*}
$$

where the spectral types are expressed in numerical values, e.g., $\mathrm{A} 0=30, \mathrm{~F} 3=43, \mathrm{G} 5=55$, and $\mathrm{K} 6=66$.

## 4. ANALYSIS

### 4.1. Local Volume Density and Total Surface Density

The method used for star count and density measurement in this survey is straightforward, while their errors are estimated using Poisson statistics, i.e., $\sigma_{N}=\sqrt{N}$, based on the incomplete star counts, which will be smaller than the complete ones and therefore provide a slightly smaller estimated error in the analysis. Our analysis is simplified at the south Galactic pole, where $b_{\text {II }}$ is approximately equal to $-90^{\circ}\left( \pm 2^{\circ}\right)$, and therefore $r \sim z$, where $z$ is the distance from the Galactic plane and $r$ is the heliocentric distance. Therefore, the density can be calculated by counting stars in each subshell or volume element as

$$
\begin{aligned}
\rho(r \simeq z) & =\Delta N / \Delta V \\
z & =|r \sin b| \simeq r, \quad \text { when } b \simeq-90^{\circ}, \\
\Delta V & =\frac{1}{3}\left(\alpha_{2}-\alpha_{1}\right)\left(\sin \delta_{2}-\sin \delta_{1}\right)\left(r_{2}^{3}-r_{1}^{3}\right), \\
\Delta V & =\int_{\alpha_{1}}^{\alpha_{2}} d \alpha \int_{\delta_{1}}^{\delta_{2}} \cos \delta d \delta \int_{r_{1}}^{r_{2}} r^{2} d r .
\end{aligned}
$$

or

$$
\Delta V=\int_{\alpha_{1}}^{\alpha_{2}} d \alpha \int_{\delta_{1}}^{\delta_{2}} \cos \delta d \delta \int_{r_{1}}^{r_{2}} r^{2} d r
$$

Star count predictions based on Bahcall's density model (Bahcall 1984a) under three different hypothesized unseen mass disks are plotted against data from this study at the SGP, as well as data from Upgren (1962) at the north Galactic pole (NGP; Fig. 7).

We used three approaches to study the density distribution of the disk stars. The first approach was carried out by taking three groups of spectral types (A8-F5 V, F6-G0 V, and G1-G6 V). The second approach was for all giants (class III) and subgiants (class IV) for which 2dF and Hydra spectra were available. The third approach also separated


FIG. 7.-Star count comparisons between the NGP and the SGP. Plus signs are from Upgren's F dwarfs at the NGP, while filled circles are from this study at the SGP. Lines are star count predictions based on Bahcall's density model under three different hypothesized unseen disks masses.
TABLE 1
Spectrophotometric, Astrometric, Kinematic, and Astrometric Data for 2615 SGP Stars

| SA No. <br> Field Pixel ${ }^{\text {a }}$ <br> (1) | $\alpha_{\mathrm{J} 2000.0}{ }^{\mathrm{b}}$ <br> (2) | $\delta_{\mathrm{J} 2000.0}{ }^{\mathrm{b}}$ <br> (3) | MK Spectral Type ${ }^{\text {c }}$ <br> (4) | Radial Velocity ( $\mathrm{km} \mathrm{s}^{-1}$ ) <br> (5) | No. of Spectra (6) | CrossCorrelation Weight <br> (7) | $m_{y}{ }^{\mathrm{d}}$ <br> (8) | $\begin{gathered} (b-y) \\ (9) \end{gathered}$ | $\begin{gathered} m_{1} \\ (10) \end{gathered}$ | $\begin{gathered} c_{1} \\ (11) \end{gathered}$ | $\begin{aligned} & \mathrm{H} \beta \\ & (12) \end{aligned}$ | No. of Obs. ${ }^{\text {e }}$ (13) | Proper Motion in R.A. ${ }^{f}$ (mas yr ${ }^{-1}$ ) <br> (14) | Proper Motion in Decl. ${ }^{\text {f }}$ (mas yr ${ }^{-1}$ ) (15) | Space Velocity $u$ ( $\mathrm{km} \mathrm{s}^{-1}$ ) <br> (16) | Space Velocity $v$ ( $\mathrm{km} \mathrm{s}^{-1}$ ) <br> (17) | Space Velocity w ( $\mathrm{km} \mathrm{s}^{-1}$ ) <br> (18) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 00000006 . | 10928.7 | -304043 | G3 III | -0.6 | 1 | 8 | 12.666 | 0.413 | 0.194 | 0.278 | 2.492 | 2 | -16.9 | -39.5 | 515.0 | -317.4 | 17.6 |
| $00000007 .$. | 11034.2 | -303100 | G6 V | 81.3 | 1 | 8 | 12.392 | 0.429 | 0.190 | 0.215 | 2.472 | 1 | 119.1 | 1.6 | -124.0 | -90.5 | -70.1 |
| 00000008 ...... | 10943.9 | -30 2940 | G2 V | 3.3 | 1 | 8 | 12.618 | 0.384 | 0.195 | 0.291 | 2.512 | 1 | -18.3 | 6.2 | 20.6 | 27.9 | -6.3 |
| 00000009........ | 10941.0 | -303000 | ... | ... | ... | ... | 14.784 | 0.340 | 0.123 | 0.355 | 2.523 | 1 | 19.6 | -12.5 | -56.7 | -137.9 | 13.6 |
| 00000010........ | 10856.3 | -3029 44 | ... | ... | $\ldots$ |  | 14.91 | 0.33 |  |  |  | 1 | 2.2 | -7.2 | 16.7 | -51.0 | 4.0 |

[^2]

Fig. 8.-Uncalibrated (top) and calibrated (bottom) correlations between spectroscopic and photometric parallaxes. For main-sequence stars of class V , the adopted transformation was $M=10.11(b-y)+0.68$ (Laird et al. 1988). For subgiants, $M(\mathrm{IV})=M(\mathrm{~V})-1.6$, and for giants of class III, $M=-1.305(b-y)+1.506$ (Lu et al. 2000).
the stars into four groups but by taking the corresponding color intervals into account. The absolute magnitudes were computed using the $b-y$ color according to Laird et al. (1988) for main-sequence stars and our study for subgiant
and giant stars (Fig. 8); the photometric groups were selected according to $M_{(b-y)}(2-3,3-4,4-5$, and 5-6). Based on their color index, the approximate spectral type groups are A5-F2, F3-F8, F9-G3, and G4-G8, respectively.

Table 2 lists the derived local volume density and total surface density together with their corresponding scale height. A detailed discussion of the vertical density distribution will be provided in $\S 5$. Our values for the scale heights are in good agreement with the thin disk model of Kent, Dame, \& Fazio (1991), which predicts a range of 200 pc , where Bahcall \& Soneira (1980) found a value of 325 pc for stars earlier than G0. Figure 9 shows the vertical density distribution perpendicular to the Galactic plane $\log \rho(z)$ versus distance $z$. Except for the F and G giants, most density distributions terminate at about 1.5 kpc for the dwarf mainsequence objects. The dashed lines represent a $\log (\rho)-z$ fit to

$$
\begin{equation*}
\rho(z)=\rho(0) \exp \left[-\left(z / z_{h}\right)\right] \tag{4}
\end{equation*}
$$

while the solid lines indicate a fit to

$$
\begin{equation*}
\rho(z)=\rho(0) \operatorname{sech}^{2}\left(z / 2 z_{h}\right) \tag{5}
\end{equation*}
$$

Overall, the sech ${ }^{2}$ function seems to fit better than the $\exp \left[-\left(z / z_{h}\right)\right]$, especially at small distances from the galactic disk. The error bars represent Poisson errors of $\sigma_{N}=\sqrt{N}$. The distribution of mass in the Galactic disk can be measured in two ways, namely, the total volume density and the total surface density. The surface density is estimated from

$$
\begin{equation*}
\sum(0)=\int_{0}^{\infty} \rho(z) d z=2 z_{h} \rho_{0} \tag{6}
\end{equation*}
$$

The total surface density can be obtained by summing the mass density at each $z$ distance. The derived total surface densities for each group are shown in the last column of Table 2. Using late-type stars of grouping 1 and 3 (G1 and later) yields an average value of $33.9 M_{\odot} \mathrm{pc}^{-2}$. This is comparable to the value of $46 \pm 9 M_{\odot} \mathrm{pc}^{-2}$ reported by Kuijken \& Gilmore (1989) but greater than $23 M_{\odot} \mathrm{pc}^{-2}$ listed for the extended halo mass for $z<1.1 \mathrm{kpc}$ (Cox 1999).

The local volume density $\rho$ can be derived from the density function and the corresponding volume. Bahcall

TABLE 2
Scale Height, Local Volume Density, and Total Surface Density

| Types | Spectral Type | Number | $\begin{gathered} z_{h} \\ (\mathrm{pc}) \end{gathered}$ | Number Density (star $\mathrm{pc}^{-3}$ ) | $\begin{gathered} \langle m\rangle \\ \left(M_{\odot}\right) \end{gathered}$ | $\stackrel{\rho}{\left(M_{\odot} \mathrm{pc}^{-3}\right)}$ | $\sum_{\left(M_{\odot} \mathrm{pc}^{-2}\right)}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Dwarf Main-Sequences Stars |  |  |  |  |  |  |  |
| Early ...................... | (A8-F5 V) | 84 | 134 | $1.03 \times 10^{-4}$ | 1.6 | $0.16 \pm 0.02$ | $4.42 \pm 0.80$ |
| Intermediate ............ | (F6-G0 V) | 509 | 210 | $1.99 \times 10^{-4}$ | 1.2 | $0.24 \pm 0.02$ | $10.30 \pm 1.26$ |
| Late ....................... | (G1-G6 V) | 496 | 240 | $7.90 \times 10^{-4}$ | 0.85 | $0.67 \pm 0.03$ | $32.23 \pm 2.16$ |
| Giant and Subgiants |  |  |  |  |  |  |  |
| III, IV ..................... | All types | 125 | 290 | $2.30 \times 10^{-5}$ | 2.5 | $0.06 \pm 0.01$ | $3.34 \pm 0.87$ |
| $M_{b-y}$ |  |  |  |  |  |  |  |
|  | (A5-F2) | 75 | 264 | $2.59 \times 10^{-4}$ | 1.8 | $0.06 \pm 0.01$ | $24.62 \pm 0.79$ |
|  | (F2-F8) | 200 | 214 | $2.33 \times 10^{-4}$ | 1.2 | $0.28 \pm 0.02$ | $11.97 \pm 1.28$ |
|  | (F8-G3) | 1130 | 306 | $5.90 \times 10^{-4}$ | 1.0 | $0.59 \pm 0.03$ | $36.11 \pm 2.75$ |
| 5-6a....................... | (G3-K0) | 732 | 251 | $8.30 \times 10^{-4}$ | 0.8 | $0.66 \pm 0.02$ | $33.33 \pm 1.51$ |

[^3]

Fig. 9.-Natural logarithm of the number density and the corresponding Poisson statistical error vs. $z$ distance for different groupings of stars. Dashed lines are fits for the exponential functions, while the solid lines are fits to the sech ${ }^{2}$ function (see text).
(1984a) found $\rho=0.2 M_{\odot} \mathrm{pc}^{-3}$ from his model in contrast to the observations of early stars in his studies that yield about $0.1 M_{\odot} \mathrm{pc}^{-3}$. The implication is that $50 \%$ of the matter is dark. The local volume density we derived from the early-type stars is $0.09 M_{\odot} \mathrm{pc}^{-3}$, agreeing with Bahcall's value and also suggesting the existence of dark matter.

### 4.2. Scale Height

The distribution of stars perpendicular to the Galactic plane varies with population and luminosity. The massive and younger (early type) stars are found relatively closer to the disk, whereas, the less massive and older (late type) stars are more dispersed to higher latitudes. Therefore, those stars that have evolved off from main sequence would have larger scale heights in comparison with the younger stars.

The variation in density with perpendicular distance $z_{h}$ (scale height) from the plane and $M$ (absolute magnitude) can be fitted by an exponential function of $z$, as given by equation (4), where $z_{h}$ is actually a function of the absolute magnitude $M$.

The scale heights listed in Table 2, which have overall errors under $15 \%$, strongly support a variation of $z_{h}$ with $M$. In four spectral type groups, including the giant stars, the scale heights are $z_{h}=134,210,240$, and 290 pc for the A8F5 V, F6-G0 V, and G1-G6 V, groups as well as for the giants and subgiants group of all spectral types. The photometric scale heights for $M_{(b-y)}$ are not monotonically increasing when compared with the spectroscopic data, probably due to small number statistics for the very early


FIG. 10.-Scale height $\left(z_{h}\right)$ vs. absolute magnitude $M_{V}$. Oort (1932) data are represented by the short dashed line, Schmidt (1963) by the long dashed line, and Upgren (1962) by plus signs, while open and filled circles are from this study for spectroscopic and photometric data, respectively.
stars. However, all values are in the same range (less than 325 pc ) as that predicted by Bahcall \& Soneira (1980).

In Figure 2 of Bahcall \& Soneira (1980), the variation in exponential scale height was plotted against $M_{V}$ for different studies. Scale heights versus absolute magnitudes in this study for both spectral and photometric data are also plotted in the same figure (see Fig. 10), and our results also fit that relation.

### 4.3. Galactic Gravitational Force $K(z)$

The astrometric data were obtained from the Yale-San Juan Southern Proper Motion (SPM) 1.0 Catalog (the SGP region; Platais et al. 1998). The spectrophotometric, astrometric, and kinematic data used in this analysis are listed in Table 1, as well as some of the spectral and photometric parallax transformations cited in § 3.5 .

Investigations of the velocity dispersion and mass density distribution were first reported by Oort (1932) using A and early F stars, which led to the suggestion of a significant missing mass in the Galaxy. Oort defined the following expression in determining the Galactic gravitational force perpendicular to the Galactic plane, $K(z)$ :

$$
K(z)=\frac{1}{\rho(z)} \frac{d}{d z}\left[\sigma_{\mathrm{w}}^{2} \rho(z)\right]
$$

where $\rho(z)$ is the stellar density at $z$ (derived in the previous section) and $\sigma_{w}^{2}$ is velocity dispersion in $z$, where in general $\sigma_{w}=\sigma_{w}(z)$.
$K(z)$ is basically a force per unit mass, or acceleration, due to the overall galactic potential at the specified location, while the density at $z$ is measured as follows:

$$
\rho(z)=\rho(0) \exp \left[\int_{0}^{z} \frac{K(z) d z}{\sigma_{\mathrm{w}}^{2}}\right]
$$

The density measurement using $K(z)$ is assumed to be isothermal only if the velocity dispersion is independent of the


Fig. 11.-Plot of $w$ velocity dispersion $\left(\sigma_{W}\right)$ vs. $z$ distance. The diamonds, plus signs, and filled circles represent early-, intermediate-, and late-type stars; the squares, crosses, and asterisks are plotted for stars in the absolute magnitude ranges 3-4, 4-5, and 5-6, respectively.
distance from the plane. A plot of the velocity dispersion versus $z$ (Fig. 11) shows no correlation for the dwarf main sequence for $z$ distances smaller than 600 pc . The larger scatter for distances greater than 600 pc is due to small number statistics.

## 5. CONCLUSIONS

In Table 2, the spectrophotometric data are divided into three main-sequence subgroups (A9-F5 V, F6-G0 V, and G1-G6 V) with MK spectra. Photometric data are separated in four subgroups in absolute magnitudes $(M=2-3$, $3-4,4-5$, and 5-6) and the giant and subgiant grouping of all types. Table 2 lists the number of stars in each subgroup, scale heights, and star counts in each subgroup. The local volume densities $\rho\left(M_{\odot} \mathrm{pc}^{-3}\right)$ and total surface densities $\Sigma\left(M_{\odot} \mathrm{pc}^{-2}\right)$ were derived using the mean mass of each corresponding type to yield the final two columns with their errors.

Our results, as derived from the early-type stars (A0-F5) and giants in spectroscopy and photometry, yield an average local volume density of $0.09 M_{\odot} \mathrm{pc}^{-3}$ compared with 0.1 $M_{\odot} \mathrm{pc}^{-3}$ observed by Bahcall (1984b), with his computed local volume density $0.2 M_{\odot} \mathrm{pc}^{-3}$, implying that dark matter exists in the galactic disk.

Using late-type stars (G1 and later), we find an average total surface density of $33.9 M_{\odot} \mathrm{pc}^{-2}$ compared with the theoretical value of $46 \pm 9 M_{\odot} \mathrm{pc}^{-2}$ reported by Kuijken \& Gilmore (1989), but larger than $23 M_{\odot} \mathrm{pc}^{-2}$ listed for the extended halo mass for $z<1.1 \mathrm{kpc}$ (Cox 1999), suggesting that there are some masses in the form of dark matter existing in the galactic disk.

The derived behavior of $K(z)$ versus distance (Fig. 12) from the galactic plane shows a well-known peak at about 400 pc . A second peak is seen at about 1100 pc , which is caused by dwarf main-sequence stars, but not by the disk


Fig. 12.-Average and individual $K(z)$ determinations vs. $z$ distance. The heavy line indicates the mean. Squares for early types, A8 $<\mathrm{SpT}<\mathrm{F} 5 \mathrm{~V}$; circles for intermediate types, $\mathrm{F} 6<\mathrm{SpT}<\mathrm{G} 0 \mathrm{~V}$; plus signs for late types, $\mathrm{G} 1<\mathrm{SpT}<\mathrm{G} 6 \mathrm{~V}$; crosses for giants and subgiants of all types; asterisks for $M_{V}=3$; triangles for $M_{V}=4$; and diamonds for $M_{V}=5$.
giants. To investigate the peak at 1100 pc in more detail, main-sequence stars were sorted into three groups of metallicities: 437 metal-poor stars with $m_{1}<0.18,493$ stars with intermediate metallicity of $0.18<m_{1}<0.21$, and 245 metal-rich stars with $m_{1}>0.21$. The $K(z)$ contribution by each group was calculated (Fig. 13). As expected that the second peak at $r \sim 1100 \mathrm{pc}$ is dominated by a thick-disk population of metal-poor objects.
The second peak is unlikely to be contaminated by a stellar cluster system such as Blanco 1 , since our survey field is located far away from the cluster. Similarly, it is not likely


Fig. 13.-Individual $K(z)$ determinations for main-sequence stars vs. $z$ distance: the circle indicates the metal-poor group with $m_{1} \leq 0.18$, the square for the metal-intermediate group with $0.18<m_{1} \leq 0.21$, and the triangle for metal-rich stars with $m_{1}>0.21$.
to be contaminated by a comoving stellar system, since we have found no such clustering in our space velocities.

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[^1]:    ${ }^{3}$ Available at http://www.ctio.noao.edu/facil/facil.html. The NOAO Web site is http://www.noao.edu/.

[^2]:    Note.-Table 1 is presented in its entirety in the electronic edition of the Astronomical Journal. A portion is shown here for guidance regarding its form and content.
    a Col. (1) uses the SGP Serial Area (SA) frame designation of pixel number in right ascension and declination and three-digit serial number in the frame.
    ${ }^{\text {a }}$ Col. (1) uses the SGP Serial Area (SA) frame designation of pixel number in right ascension and declination and three-digit serial number in the frame.
    ${ }^{\text {c }}$ Col. (4) lists the MK spectral type using the 2 dF or Hydra spectra. A spectral type preceded by a colon or absence of luminosity class is generally referred to as uncertain class pixel or column, an " $n$ " when an image near an edge or a corner of the CCD frame, and a colon when the measures in magnitude or colors are uncertain. ${ }^{\mathrm{f}}$ Columns list proper motion adopted from Yale-San Juan SPM catalog (Platais et al. 1998).

[^3]:    a Values are $M_{b-y}$ not types.

